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GRAMMAR SCHOOL ALGEBRA

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Received May 15, 1903.

Grammar-School Algebra

SEVENTY-FIVE SUGGESTIVE LESSONS
FOR BEGINNERS

BY

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PREFACE.

While preparing this little manual for the grammar-school children who may use it, the author has had for his guide the well-known pedagogical principle, "The primary concepts and ideas in every branch of knowledge should be taught objectively in all grades of school." There has seemed to him to be too sudden a transition from the numerical to the literal symbol. So much explanation, too, is the rule that children from the start gain an idea that algebra is an *alias* for mystery. While it must be admitted that there are many new truths to be learned by the pupil in its study, it should not be forgotten, on the other hand, that it contains much that is in no way contradictory to what he has already learned, and which, if given a chance, with no hint that it is something new, he can use in another form.

The author has for some time felt that there should be a difference, other than in thickness, between the book prepared for grammar-school children and the book prepared for the high-school pupil. The expression $(x+y)^2 = x^2 + 2xy + y^2$, for example, should mean something more to a child than the letters of the alphabet

which the answer contains. It can be made to do so if the teacher will make an effort to present proper conditions for its application *to things*. A study of the lessons here presented will show that such an effort has been made by the author; how effectually he will not presume to say, but will be pardoned if he expresses a hope that the little work will meet with the approbation of those into whose hands it may find its way.

WILLIAM M. GIFFIN.

COOK COUNTY NORMAL SCHOOL,
CHICAGO, 1895.

TO THE TEACHER.

The author has purposely avoided the exercises so often given in most algebras, viz., find the value of x in the following equation, $x + 2 = 12$, for the reason, that, in his judgment, such exercises are not productive of mental growth. The child has simply to recall certain facts already learned by rote to work the equation *made for him*. On the other hand, if he be given the conditions, viz., a certain number increased by two equals twelve, he not only has to recall the facts as before, but he has to draw an inference and through this inference to *form his own equation*, which becomes a part of himself, thus enhancing its value and causing its solution to be far more pleasurable. We must not forget that "teaching is but the presentation of external conditions for educative self-effort. Thinking is not in itself educative; it becomes educative only when the conscious action is intense, and the conscious activities are immediately needed for development." *

On pp. 99–108 are found many exercises which, though simple in themselves, are much more valuable

* Talks on Pedagogics.

than the isolated equations referred to. It is thought they will hardly be necessary for those children who are quick to grasp new principles, while others who are slower may require many repetitions. They may be used at the time the pupils first take up the subjects or as a final review.

Only such factoring and fractions are given as are required for the child's immediate use. For this reason, also, nothing has been done with the "highest common divisor," the "lowest common multiple" (as such), or quadratic equations. In short, the book is not a high school book, but a preparation for the high school book.

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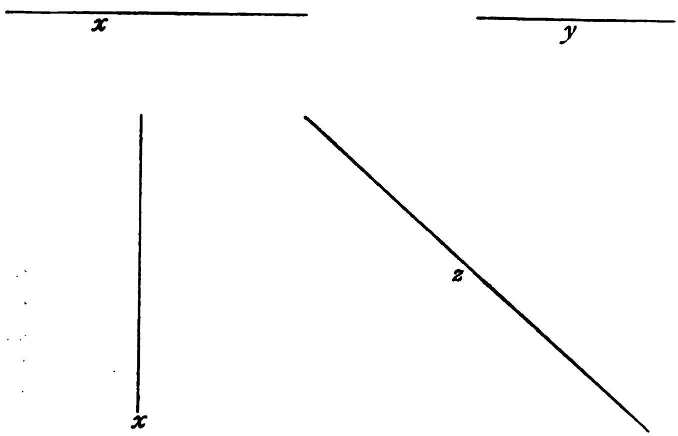
CHART I.



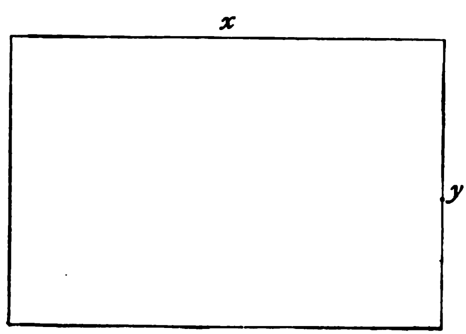


FIG. 1.

INTRODUCTORY LESSONS.

LESSON I.

1. How long is a line that equals the two horizontal lines at the top of Chart I?

2. How long would the longer horizontal line be if a line equal to the shorter were taken from it?

3. How long is a line that equals the longer horizontal and the vertical lines? Write your answer in two ways.

4. How many lines equal to the shorter horizontal line could be made from the oblique line?

5. The shorter horizontal line is equal to what part of the oblique line?

6. How long is a line half as long as the vertical and oblique lines together?

LESSON II.

CHART I.

7. Which two lines (Chart I) equal $x + y$?

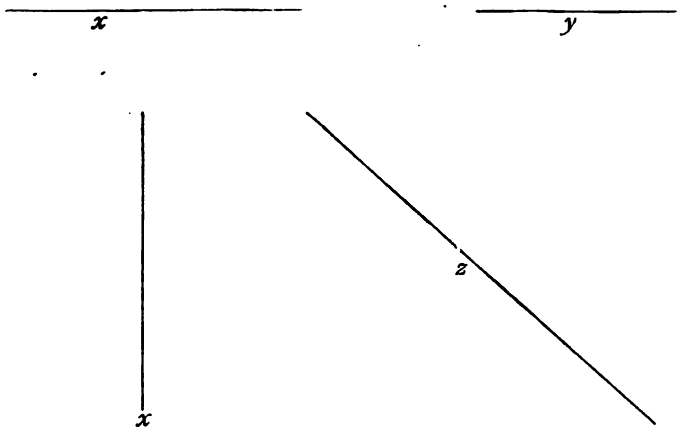
8. Which line would equal $x - y$?

9. Which two lines equal $2x$?

NOTE.—Teach coefficient here. Speak and write the word on the blackboard at the same time, erase, and then have the pupils write it. A coefficient shows how many times a quantity is taken. When no coefficient is expressed 1 is always understood. 5 times $x = 5x$, just as 5 times \$1 = \$5. Or 5 times one hat = 5 hats.

10. What would equal $z \div y$, or $\frac{z}{y}$?

CHART I.



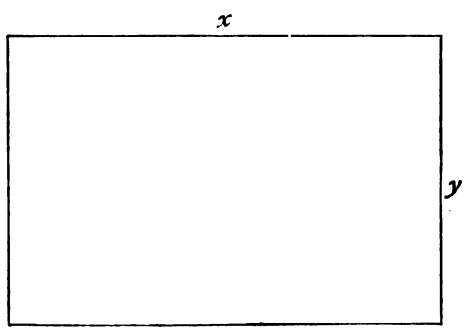


Fig. 1.

11. The expression $\frac{y}{z}$ is an answer to what question in Lesson I?

12. To what question is $\frac{1}{2}$ of $(x + z)$ an answer?

LESSON III.

CHART I.

13. Express the sum of x and y.

14. Express the difference between x and y.

15. Express the product of x and y.

NOTE.—xy is preferable to $x \times y$, a times $b = ab$ rather than $a \times b$, and a times b times $c = abc$.

16. Express the quotient of x and y.

NOTE.—In Algebra the form $\frac{x}{y}$ is more often used than $x \div y$.

17. Express $\frac{1}{2}$ the sum of x and y.

18. Express $\frac{1}{2}$ of x, plus y.

19. Which is larger, $\frac{1}{2}$ of $(x + y)$ or ($\frac{1}{2}$ of x) plus y?

NOTE.—If there is a difference of opinion give values to x and y as 6 and 4.

LESSON IV.

20. See Fig. 1, Chart I, to which these questions refer. Express its perimeter in three ways. (The perimeter is the sum of all of its sides.)

21. y is what part of the perimeter?

22. x is what part of the perimeter?

23. What is half of its perimeter?

24. What does $(2x + 2y)$ express?

NOTE.—Read, the quantity two x plus two y.

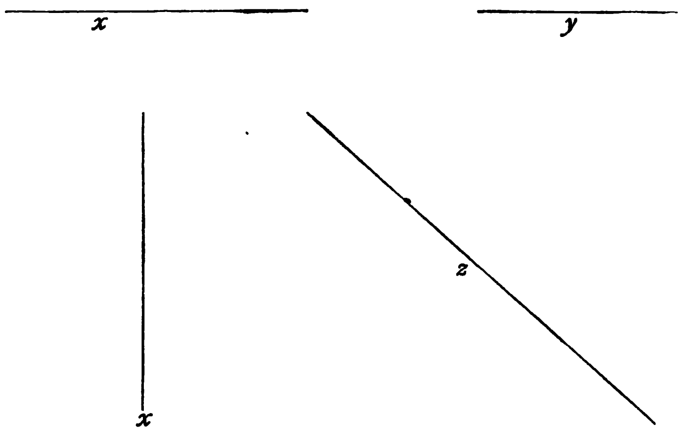
25. What does $2(x + y)$ express?

NOTE.—Read, two times the quantity x plus y.

26. What does $\frac{y}{2x + 2y}$ express?

27. What does $\frac{2(x + y)}{2}$ express? What $x + y$?

CHART I.



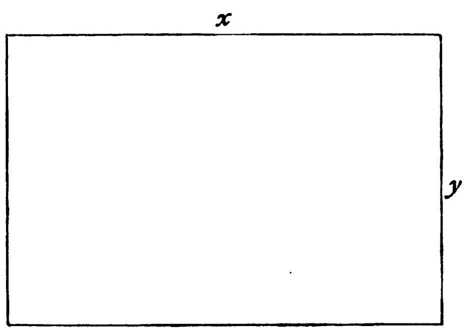


FIG. 1.

28. What does $\frac{x+y}{2}$ express?

29. What does $x + y + x + y$ express?

30. Express $\frac{1}{6}$ of the perimeter of Fig. 1 in two ways.

LESSON V.

NOTE.—The first five questions refer to Fig. 1, Chart I.

31. What is the area of Fig. 1?

32. What does $\frac{x+y}{3}$ express?

33. What does xy express?

34. Express $\frac{1}{8}$ of the perimeter.

35. What does $\frac{xy}{2}$ express?

36. Express $\frac{1}{3}$ of the area of Fig. 1.

37. Express the sum of the two horizontal lines added to the sum of the vertical and oblique lines. (See the four lines above Fig. 1, Chart I.)

38. Express 4 times the sum of the vertical and oblique lines.

39. What does $(x + y) + (x + z)$ express?

NOTE.—Many original questions may be given if desired.

LESSON VI.

40. Express the perimeter of Fig. 2 in three different ways.

41. What is a rectangle called whose area contains just as many rows of square units as there are square units in a row?

42. When told to square 2, of what is one to think?

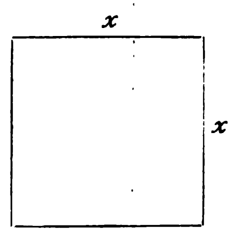
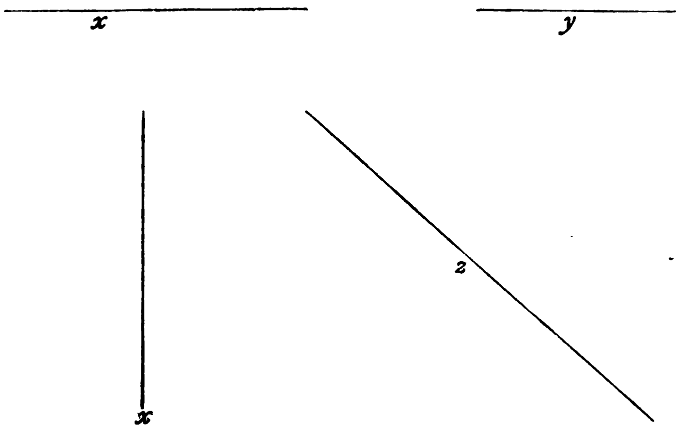


FIG. 2.

NOTE.—Of a square containing 2 rows of 2 square units in a row. Hence I am to multiply 2 by 2, which equals 4. Four is the square of 2.

CHART I.



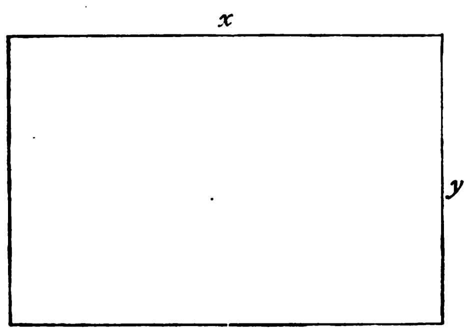


FIG. 1.

43. How shall one find the area of Fig. 2?

44. What is the area of Fig. 2?

NOTE.—To find the area we multiply x by x and get xx or x^2. The small figure above and to the right is called an *exponent*. When no exponent is written 1 is understood. Thus, $x^1 = x$. An exponent shows what power of the number is to be obtained, or how many times it is to be taken as a factor. It is written at the right and above the quantity as x^2.

45. In question 44 what was the exponent of x in the multiplicand?

46. What was the exponent in the multiplier? What was the exponent in the product?

47. How does the exponent of the product compare with the exponents of the multiplicand and multiplier?

NOTE.—Multiplicand is x^1. Multiplier is x^1. Product is x^2.

48. What would be the exponent of x^2 multiplied by x, of x by x^2, of x^3 by x^2?

49. What does x^2 express about Fig. 2?

50. What does $\sqrt{x^2}$ express about Fig. 2?

LESSON VII.

51. Express $\frac{1}{3}$ the perimeter of Fig. 2.

52. Express $\frac{1}{4}$ the perimeter of Fig. 2.

53. Express $\frac{1}{8}$ of the perimeter of Fig. 2 in two different ways.

54. What part of the perimeter of Fig. 2 does $\frac{x}{3}$ express? $\frac{x}{4}$? of $\frac{2\,x}{2}$?

55. What part of the perimeter of Fig. 2 does $3\,x$ express?

56. What part of the perimeter of Fig. 2 does $\frac{3x}{2}$ express?

57. Express the square of x; of y; of z.

58. Express the cube of x; of y; of z.

59. How long is each side of a square whose area is y^2? whose area is z^2?

LESSON VIII.

60. How many terms are used to express the area of Fig. 2?

When a quantity consists of only one term it is called a *monomial.* Close your books. Write *monomial.* Write a monomial using 2 and y; 3 and x and y.

61. How many terms are used to express the sum of the two horizontal lines on Chart I?

When a quantity consists of only two terms it is called a *binomial.* Write the word *binomial* without looking at the word in your book. Write a binomial using y and z; x, y and z.

62. How many terms are used to write the sum of the two horizontal and oblique lines on Chart I?

When a quantity consists of only three terms it is called a *trinomial*. Write the word. Write a trinomial using 2, x and z; -2, x, z and y.

When a quantity consists of more than *two* terms, it is called a *polynomial.*

63. When a quantity consists of two terms what kind of a quantity is it?

64. Write a *binomial* representing the sum of the two horizontal lines, and the vertical line on Chart I.

65. Write a *trinomial* representing the sum of the same three lines.

LESSON IX.

66. From the oblique line (Chart I) take the sum of the two horizontal lines.

67. How many terms are used to represent your answer?

NOTE.—Only two, since the symbol ( ), called a *parenthesis*, is used to enclose two terms that are to be taken together, as one term. Thus, $z - (x + y)$. Read, z minus the quantity x plus y. A parenthesis is also used when two or more terms are to be taken together as one *factor*. Thus, when we expressed the perimeter of Fig. 1, the shortest way, we had $2 (x + y)$. Read, 2 times the quantity $x + y$.

68. Show that the difference between the vertical and shorter horizontal lines is taken from the oblique line Chart I.

69. In the *monomial* $4 x^2$, what do we call 4? What do we call 2? *Let each one write the answers.*

70. Write all the exponents found in the following *polynomial:* $4 x^2 + 2 y^3 + 6 z^2 + y$.

71. Write all the coefficients found in the following *trinomial:* $5 y^2 + 8 y z^3 - z$.

LESSON X.

If x and y represent two numbers, x being the greater, what will represent—

72. Their sum?
73. Their difference?
74. Their product?
75. Their quotient?
76. The part of x to which y is equal?
77. The square of their sum?
78. The sum of their squares?
79. The cube of their sum?

80. The sum of their cubes?

81. The product of their sum and difference?

82. Their product times their difference?

83. The quotient of their sum and difference?

84. What is the *sum* of 4 and x?

85. What is the *product* of 4 and x?

86. What does $4 + x$ represent?

87. What does $4x$ represent? $(x+y)(x-y)$?

LESSON XI.

EXERCISE IN THE USE OF ALGEBRAIC LANGUAGE.

88. Henry had x cents and Will gave him y cents. How many cents did he then have?

89. John had 4 dollars and earned x dollars more. How many dollars did he then have?

90. Grace had x dollars and Jennie had 4 times as many dollars. How many dollars did Jennie have?

91. Mabel had x dollars and spent 2 dollars. How many dollars did she have left?

92. John had x dollars and divided them among y boys. How many dollars did each boy get?

93. Fred had x dollars and bought hats at y dollars each. How many hats did he buy?

NOTE.—$\frac{1}{y}$ of x for Ex. 92, and $\frac{x}{y}$ for Ex. 93.

94. A rectangle is x feet long and y feet wide. What is its perimeter?

95. What is the area of the rectangle? See problem 94.

96. What is the area of a square whose sides are y feet long?

97. I bought a horse for $50 and sold it for x dollars. How much did I gain?

98. William had $17a$ quarts of berries and ate $2a$ quarts. How many quarts did he still have?

LESSON XII.

99. I bought a whip for c cents and gave the merchant a 50¢ piece. How much change did I receive?

100. What will be the cost of 3 hats at x dollars apiece and 5 collars at y cents apiece?

101. A man bought y books at \$5 each and sold them at \$8 each. What did he gain?

102. I have some picture wire x yards long. If from this I cut 5 pieces each y feet long, how many feet will I have left with which to hang pictures?

103. I have a triangle with its base x feet long and its altitude $2y$ feet long. What is its area?

NOTE.—Picture the triangle as a rectangle with the same base as the triangle and an altitude ½ that of the triangle. See "Supplementary Arithmetic" by the author.

104. Mr. Johnson has a lot containing y acres for which he asks \$500. How much is that an acre?

105. 6 oranges cost me $5y$ cents. What will 10 oranges cost me at the same rate?

106. At x dollars apiece, how many chairs will y bushels of wheat buy at z dollars a bushel?

107. The shorter parallel side of a trapezoid is b feet long. The longer parallel side is c feet long. The altitude is $2d$ feet long. How many square feet in its area?

NOTE.—Picture the trapezoid as a rectangle with a base = to the sum of the parallel sides and an altitude ½ that of the trapezoid.

108. At x cents each how many peaches will \$2 buy?

LESSON XIII.

109. The diameter of a circle is $2x$ feet. Its circumference is $2y$ feet. What is its area?

NOTE.—Picture the circle as a rectangle with base ½ the circle's circumference and an altitude ½ the circle's diameter.

110. How many square feet on the four walls of a room y feet long, x feet wide and z feet high?

NOTE.—Picture the room as a rectangle with a base equal to the perimeter of the room and an altitude equal to the height of the room.

111. A cylinder is x feet in circumference and has an altitude x feet long. What is the area of its convex surface?

NOTE.—Picture the cylinder as a rectangle with a base equal to its circumference and of the same altitude.

112. At $8 apiece how many sheep will y dollars buy?

113. Mr. White bought a farm of x acres at y dollars an acre and sold it at z dollars an acre. How much did he gain by the transaction?

114. How many board-feet in 18 boards, each 12 feet long and x inches wide?

NOTE.—A board 12 feet long contains as many board-feet as it is inches wide.

115. How many board feet in 30 boards, each 10 feet long and x inches wide?

116. If x men can do a piece of work in y days, how long will it take z men to do it?

117. John has $5, his father gives him $3 and he earns x dollars. How much has he altogether?

118. He then spends $4. How much has he left?

LESSON XIV.

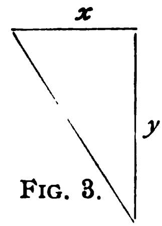


FIG. 3.

119. What is the perimeter of Fig. 3?

120. What represents $\frac{1}{2}$ of its perimeter?

121. Represent the perimeter of the rectangle to which it is equal.

122. Which is longer, the perimeter of the triangle or the perimeter of the rectangle to which it is equal?

123. Represent the triangle's area; $\frac{1}{2}$ its area.

124. What does $x + y + \sqrt{x^2 + y^2}$ represent? (See Fig. 3.)

125. What will represent the perimeter of a right-triangle whose base is a feet long and whose altitude is b feet long?

126. What will represent the perimeter of a right-triangle with an altitude $4x$ feet long and a base $3x$ feet long?

127. What will represent the area of the last triangle? $\frac{1}{2}$ its area? $\frac{1}{4}$ its area?

128. Represent the altitude of a right-triangle whose base is $4y$ feet long and whose hypotenuse is $5x$ feet long.

LESSON XV.

ADDITION.

In algebra we have to deal with what are known as *positive* and *negative* quantities. They have directly opposite tendencies in mathematical calculations. The sign + is prefixed to positive quantities, and the sign − is prefixed to negative quantities.

129. If we call the gain in business *positive*, what shall we call the loss?

130. If the direction east is called positive, what shall we call west?

131. Tell which of the following quantities are positive and which negative: Mr. A deposits \$8 in the bank, "checks out" \$5, "checks out" \$2, puts in \$15, deposits \$80, "checks out" \$16.

132. Which of the following: Henry walks 5 miles east on Monday, 2 miles east on Tuesday, 3 miles west on Wednesday, 2 miles west on Thursday. 8 miles east on Friday and 6 miles west on Saturday.

133. Write the following quantities with their proper sign: Mr. Jones buys $14x$ books, sells $3x$, buys $8x$, buys $7x$, sells $2x$, sells x, sells $4x$, buys $18x$.

134. What was the combined effect of the above transactions upon the number of books Mr. Jones had on hand?

$$\begin{array}{r} +14x \\ -3x \\ +8x \\ +7x \\ -2x \\ -x \\ -4x \\ +18x \\ \hline +37x \end{array}$$

SOLUTION.—The positive quantities combined equal $+47x$ books; the negative quantities combined equal $-10x$ books. Subtracting the negative quantities from the positive, we obtain $+37x$ books. This result is called the *Algebraic Sum*.

135. What do you discover the *Algebraic Sum* is the result of?

Name at sight the algebraic sums of the following: When no sign is given + is always understood.

136.	137.	138.	139.	140.	141.
$-3x$	$+3x$	$7x$	$-7x$	$-7x$	$-6x$
$+9x$	$-9x$	$3x$	$-3x$	$+3x$	$+6x$

LESSON XVI.

Find the algebraic sums of the following:

142.	143.	144.	145	146.	147.
$28xy$	$-9ab$	$+64z^2$	$+19x$	$45x^2y$	$-98cd$
$16xy$	$-8ab$	$+16z^2$	$+18x$	$18x^2y$	$-98cd$
$32xy$	$-7ab$	$+19z^2$	$-17x$	$-18x^2y$	$-88cd$
$-11xy$	$+8ab$	$+8z^2$	$+16x$	$-45x^2y$	$-10cd$

148. $16xyz + 16xyz - 16xyz - 16xyz$.

LESSON XVII.

Name the algebraic sums of the following:

149.	150.	151.	152.	153.
$+x$	x	$-y$	xy	a
$+y$	x	$-y$	xy	b
$-z$	$-y$	$-x$	$-xy$	$-c$

What is the algebraic sum of each of the following:

154.	155.	156.	157.	158.	159.
$-3x$	$2xy$	$-6z$	$-8xyz$	x	$-y$
$3x$	$3xy$	$+6z$	$+8xyz$	$-x$	$+y$

(*a*) What is the sum of $6xy + n + 5ax + 10am - 6xy + 6n - 6ax - 8am, + 8xy - 9n + 3ax + n - am$.

(*b*) What is the sum of $8xyz + 3xy^2 - 6abc - 9axy + 23xy^2 - 4abc - 7xyz - 3xy^2 + abc$.

(*c*) Add $3x^2y^2 + 2xy - 3ax$ to $4ax - 2x^2y^2 - 16xy$.

(*d*) Add $7x^2b - 3xbc - 8b^2c - 9c^3 + cd^2$, $8xbc - 3x^2b - 9b^2c + 14c^3 - cd^2$.

NOTE.—See page 109 for explanation of process. Many more examples may be given if the teacher thinks best.

LESSON XVIII.

SIMPLE EQUATIONS.

160. What do we call such an expression as $3 + 2 = 5$?

NOTE.—An equation, if true, must always balance. If I have a one-pound weight on one end of a pair of scales, how many ounces must I place on the other end to make the scales balance? If we have $8 + 2$ for the *first member* of an equation, what must be the *second member?* An equation, like the scales, must, to be true, always balance.

Complete the following equations:

161.	162.	163.	164.	165.	166.
$3 + 4 =$	$6 - 2 =$	$12 + 4 =$	$= 3 + 5$	$= 8$	$10 =$

NOTE.—All terms at the left of = are the first member. All terms at the right of = are the second member. A member may consist of one or more terms. Tell which member was wanting in each of the above equations.

In the following equations what must x equal?

167.	168.	169.
$x = 3 + 2$	$x = 12 - 4$	$x = 6 + 4$

170. John had a certain number of marbles and Henry had 3 times as many. Together they had 20. How many marbles had each?

Let x equal the number John had.

And $3x$ equal the number Henry had.

Then will $x + 3x = 20$.

Therefore $4x = 20$,

And $x = 5$. The number John had.

$3x = 15$. The number Henry had.

171. The sum of two numbers is 24. The greater number is two times the less. What are the numbers?

172. Mr. White bought a hat and a coat, paying 5 times as much for the coat as for the hat. What was the cost of each if both cost him $18

173. A certain number added to 8 times itself equals 27. What is the number?

174. My watch and chain together are worth $80. The chain is worth but $\frac{1}{3}$ as much as the watch. What is the value of each?

NOTE.—Avoid fractions here.

175. Mr. G. is three times as old as his son Cleon. The sum of their ages is 60 years. Find the age of each.

176. The perimeter of a rectangle is 24 inches. The vertical sides are just $\frac{1}{3}$ as long as the horizontal sides. How long is each side?

177. The perimeter of a square is 24 inches. What is the length of each side?

178. A and B went into business together with a cash capital of $800, A putting in 4 times as much as B. How much did each man invest?

179. The sum of what two numbers equals 132, if one of the numbers is ten times as great as the other?

180. John had a certain number of apples. Roy had 2 times as many as John, and Harry had as many as John and Roy together. The sum of their apples was 36. How many apples had each?

LESSON XIX.

181. Look at equation (a). What was done to both of its members to get equation (b)?

$$(a)\ 6 = 6$$
$$(b)\ 8 = 8$$

182. Was the equality destroyed?

183. What equation would have been formed had both members been *multiplied* by 2?

184. Would the equality have been destroyed?

185. What equation would have been formed had 2 been *subtracted* from both members?

186. Would the equality have been destroyed?

187. What equation would have been formed had both members been *divided* by 2?

188. Would the equality have been destroyed?

189. Had both members been increased, diminished, multiplied or divided by any other number than 2, would the equality have been destroyed?

190. Copy or read and fill the blanks: If the same or equal quantities be added to or subtracted from______ members of an______the______is not destroyed.

NOTE.—Take time to fill in the blanks. There is no reason for haste.

191. Copy or read and fill the blanks: If______members of an equation be multiplied or divided by the same or______quantities the equality is not______.

LESSON XX.

192. B had a certain number of dollars. After spending \$8 he had \$24 left. How many dollars had he at first?

Let x = the original number.
Then will $x - 8 = 24$.

We cannot find the value of x until we get rid of 8 in the first member. This we can do by adding $+ 8$ to both members of the equation, thus:

$$\begin{array}{rcl} x - 8 &=& 24 \\ +8 && +8 \\ \hline x &=& 24 + 8 \\ x &=& 32 \end{array}$$

193. If 6 be taken from 3 times a certain number the difference will equal 33. What is the number?

194. Take 12 from 4 times a given number and 68 will be left. What is the number?

195. Jackson gave Edith 18 of his marbles and he then had 80. How many had he at first?

196. George and Charles together have 42 cents, and Charles has 6 cents less than George. How many has each?

197. Willie's money added to Henry's equals 41 cents, but Henry has 9 cents more than Willie. How many cents has each boy?

NOTE.—Let x = the number of cents Willie had and $x + 9$ = the number of cents Henry had, and we have $x + x + 9 = 41$, and $2x + 9 = 41$. To get rid of 9 in the first member we add -9 to both members. By careful inspection we see that if we write the equation $2x + 9 = 41$ and omit the 9 in the first member and write 9 in the second member, with its sign changed thus, $2x = 41 - 9$, we obtain the same result. This operation is called *Transposition*, *i.e*, we say we transpose 9 from the first to the second member after changing its sign. What we really do is to add -9 to both members. We shall often have occasion to "transpose" a term from one member to another; hence, it is well to become familiar with the operation.

198. Find the value of x in this equation: $3x + 8 - 4 = 32 - x$.

NOTE.—Here we desire to get rid of $+8$ and -4 in the first member, and of $-x$ in the second member. This we can do by transposition, thus: $3x + x = 32 - 8 + 4$, then $4x = 28$ and $x = 7$.

199. What did we really do to both members of the last equation?

NOTE.—We added $+?$, $-?$ and $+?$ to both members of the equation.

200. Find the value of x in the equations here given:

$5x + 10 = 34 - x$. $2x + 4 = x + 6$. $2x + 1 = x + 5$.

201. If to 5 times John's money we add $10, the sum will equal $34 less John's money. How much money has he?

202. If to 7 times a certain fraction we add 5, the sum will equal 9 less 4 times the fraction. Find the fraction.

203. If to 7 times my baby's age you add 5 years, you will have my age, which is 26. How old is the baby?

LESSON XXI.

204. Thomas sells morning papers. On Monday he made a profit on his sales. On Tuesday he made 3 times as much, and on Wednesday he made 8 cents, when he found his profits for the three days amounted to 48 cents. How much did he make each day?

205. Mr. Holmes has a journey of 120 miles to walk. Tuesday he walks twice as far as on Monday, and Wednesday he walks as far as on the other two days together, when he reaches his destination. How far did he walk each day?

206. If from three times a given number three be taken, and 10 be added to the difference, a number equal to 14 plus 2 times the given number will be obtained. Find the number.

207. If to the sum of two numbers, one of which is 4 times as large as the other, we add twelve, we shall have a number equal to the number of quarts in a bushel. What are the numbers?

208. A boy who was 16 years old asked another boy how old he was, when the other boy said: "If to my age you add 8 you will get a number $\frac{1}{8}$ greater than your own age." How old was he?

209. Two bins contain 55 bushels of grain, and the larger one contains twice as much as the other lacking 5 bushels. How many bushels in each bin.

210. If from 2 times a certain number I take 7, the difference will be one greater than the number. Find the number.

211. Find the fraction which, if multiplied by 10, will give a number equal to 5, less 8 times itself, plus 3.

212. If I were to add 8 times my money to \$7, I would have the same amount that I would have were I to add \$39 to 4 times my money. How much money have I?

213. A and B each took 5 bushels of apples to market for which they both received \$16. B received 10¢ more per bushel than A. Find what each received.

LESSON XXII.

Copy or read and fill in the blanks. If both members of an equation be multiplied by the same or.........quantities the.........will not be.........

214. A and B together sold 7 bushels of apples. B sold as many as A and $\frac{1}{3}$ as many more. How many bushels did each sell?

Let x = the number A sold.

And $x + \frac{x}{3}$ = " B "

Then will $x + x + \frac{x}{3} = 7.$

Getting rid of the fraction we have

$$3x + 3x + x = 21.$$

215. What did we do to get rid of the fraction?

NOTE.—When we multiply both members of an equation containing fractions, by the common denominator, the fractions are changed to integers. The operation is called *Clearing the Equation of Fractions.*

216. Three boys, A, B and C, had altogether \$21.25. B had $\frac{3}{4}$ as much as A, and C had $\frac{1}{3}$ as much as B. How many dollars did each boy have?

217. 60 acres of land were divided among three children so that the second received twice as much as the first and the third $\frac{1}{4}$ as much as the other two. How many acres did each child get?

218. What number will equal 2 if from its $\frac{1}{3}$ its $\frac{1}{4}$ be taken?

219. George's father bought him a horse. When George asked him what he paid for it he laughingly said: "If to $\frac{1}{3}$ of twice its cost you add $\frac{1}{4}$ its cost, and from the sum take $\frac{1}{6}$ of 4 times its cost you will get \$7. Now you may tell me what I paid."

220. If to two times my money I add \$2 and take $\frac{1}{3}$ of the sum, it will equal \$4 added to $\frac{1}{4}$ of my money. How many dollars have I?

221. Find the value of x in the following equation: $\frac{2x+4}{6} - \frac{1}{6} = \frac{x}{4} + 2$. Write an original problem for this equation.

222. If $\frac{1}{4}$ of 7 times a certain fraction be diminished by the fraction $\frac{5}{6}$, it will be equal to $\frac{1}{12}$ of 5 times the same fraction. What is the value of the fraction?

223. To what number can we add 4 and have $\frac{1}{3}$ the sum equal to $\frac{1}{6}$ of twice the same number added to 3?

LESSON XXIII.

224. Two numbers whose sum is 50 are to each other as 2 is to 3. Find the numbers.

Let x equal the less number, then we have a proportion $x : ? :: 2 : 3$, from which we find that the greater number is $\frac{3x}{2}$.

$$\text{Hence, } x + \frac{3x}{2} = 50.$$

225. How did we find that the greater number was $\frac{3x}{2}$?

226. Two numbers are to each other as 4 is to 6. Their sum is equal to 5. Find the numbers.

227. Find two numbers which are to each other as 8 is to 4, and whose sum is 18.

228. A and B together caught 36 fish. A's number was to B's as 9 is to 3. How many did each catch?

229. There are three numbers whose sum is 170. The first is to the second as 2 is to 3, the third is 4 times the second. What is the value of each number?

230. Divide 48 into two parts that shall be to each other as 12 is to 4.

231. Divide 24 into three numbers so that the first will equal $\frac{1}{2}$ the third and the second will equal the sum of the first and third.

232. John, Will and James had \$7. Will's share equaled $\frac{1}{2}$ of John's, and James' share was equal to $\frac{1}{2}$ of Will's. How much had each boy?

233. Myra and Jennie had \$82. 2 times Myra's money, less \$8, equaled Jennie's money. Find how much each girl had.

234. Seven times a certain number, diminished by 4 times itself, equals 63. What is the number?

LESSON XXIV.

SUBTRACTION.

Add at sight the following:

235.	236.	237.	238.	239.
$-3x$	$-2y$	a	$-z$	xy
$2x$	$-4y$	b	x	xy
$4x$	$6y$	$-c$	y	$-xy$

240. George had x apples and gave John y apples. How many had he left?

(*a*)		ADD. (*b*)
x	Minuend.	x
y	Subtrahend.	$-y$
$x-y$	Difference.	$x-y$

241. When we subtracted y from x in example (*a*) how did the answer compare with the answer in (*b*), where we added $-y$ to x?

242. Then what might we have done in example (*a*) to get the same answer?

243. What numbers may x and y represent?

244. Since x and y may represent any numbers, what have we discovered may always be done when we subtract one quantity from another?

245. George has x apples and gives John $y-z$ apples. How many apples has he left?

(*c*)		(*d*) ADD.
x	Minuend.	x
$y-z$	Subtrahend.	$-y+z$
$x-y+z$	Difference.	$x-y+z$

NOTE.—If George had x apples and gave $y - z$ apples to John we may say $x - y + z$ is what he had left. Because when we say $x - y$ we see at once that we have taken away too much by z. Hence, our answer is z too small. If we add z, then, our answer is correct, which is $x - y + z$.

246. When we subtracted $y - z$ from x in example (c), how did the answer compare with the answer in (d) where we added $-y + z$ to x?

247. Then what might have been done in example (c) to get the same answer?

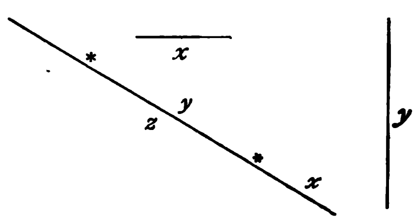
248. Copy or read and fill in the blanks. In algebraic subtraction we change the signs of the______, and then find the algebraic sum.

NOTE.—To be more sure that we are right in our conclusion we will make still another test. Take the example:

	8	Minuend.
	6	Subtrahend.
6 from 8 equals what?	2	Difference.

The minuend is 8, the subtrahend is 6, and the difference is 2. We learned in arithmetic that the difference added to the subtrahend equals the minuend. This is true, as we find that by adding 2 to 6 we obtain 8.

Now we will take example (a): The minuend is x, the subtrahend is y and the difference is $x - y$. Adding the difference $x - y$ to the subtrahend $+ y$, we obtain x the minuend. Find whether this is true in example (c):



249. How long will the oblique line be if we take from it a line equal to the sum of the horizontal and vertical lines?

NOTE.—It is evident that it will be the difference between z and $x+y$ or $z-x-y$ long. See cut. It is just as evident that if we add to what is left of z, viz.: $z-x-y$ (the difference),
what we took away, viz.: $x+y$ (the subtrahend),
we obtain the original line z (the minuend).

Name at sight the algebraic differences between the following:

250.	251.	252.	253.	254.
$6x$	$8y$	$-8y$	$8x$	$12xy$
$2x$	$3y$	$3y$	$-3x$	$4xy$
$4x$				

255.	256.	257.	258.	259.
$12xy$	$9xyz$	$3x$	$4x^2$	$11x^2y^2$
$8xy$	$-4xyz$	$-9x$	$7x^2$	$-7x^2y^2$

LESSON XXVI.

260. What is the perimeter of Fig. 7? [$2x+2y+2y$. Answer.]

261. If one end be taken from the perimeter, how much of it will be left? [$2x+2y+y$. Answer.]

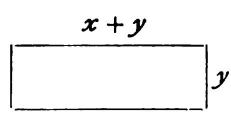


FIG. 7.

NOTE.—Your work should be written out thus:

$$\begin{array}{r} 2x+2y+2y \\ +\ y \\ \hline 2x+2y+\ y \end{array} \quad \text{or} \quad \begin{array}{r} 2x+4y \\ y \\ \hline 2x+3y \end{array}$$

262. If the two ends be taken from the perimeter, how much of it will be left?

263. If one horizontal side be taken from the perimeter, how much of it will be left?

264. How much of the perimeter will be left if the two horizontal sides be taken?

265. How much of the perimeter will be left if both horizontal sides and one end be taken?

NOTE—(1) Your work should be written out in full. (2) Inspect the figures each time and satisfy yourself that your answer is correct. (3) Thus is seen the truth of the principle as to changing the sign of the subtrahend.

LESSON XXVII.

266. What is the perimeter of Fig. 8?

267. How much of the perimeter will remain if the shorter parallel side and the oblique side are both taken from it?

[See Note under Ex. 265]

268. If the oblique side and a line 3 times as long as the shorter parallel side be taken from the perimeter, how much of it will remain.

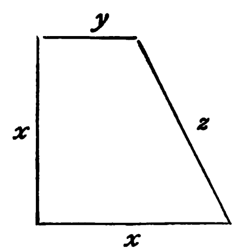


FIG. 8.

NOTE.—Give values to the letters and see if your answer $2x - 2y$ is correct. $x = 2$ inches, $y = 1$ inch and $z = 2\frac{1}{4}$ inches.

269. Find how much of the perimeter will be left if the shorter horizontal side and a line 2 times as long as the longer parallel side be taken from it.

NOTE.—Remember, your work should be written out in full. Add many questions to those given by the author. Write original questions for your teacher.

LESSON XXVIII.

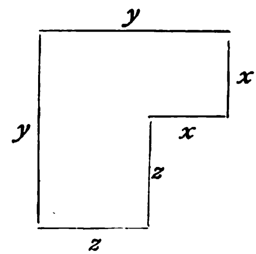


FIG. 9.

270. What is the perimeter of Fig. 9?

Give the shortest answer.

271. Subtract the shortest horizontal side from the perimeter.

272. From the perimeter take all of the vertical sides. What is left?

273. Take from the perimeter the two shorter and the two longer sides.

274. Take from the perimeter a line equal to the difference between the two longer vertical sides.

NOTE.—Give values to the letters and test this answer: $x = 1$ $y = 2\frac{1}{2}$, $z = 1\frac{1}{2}$. Also inspect the figure.

275. From the perimeter take a line equal to the difference between the shortest vertical and longest horizontal sides. What remains?

$$\begin{array}{r} 2x + 2y + 2z \\ -\ x + \ y \\ \hline 3x + \ y + 2z \end{array}$$

276. How much of the perimeter will be left if there be taken from it a line equal to the difference between 2 times the longest vertical side and 2 times the shortest horizontal side?

NOTE.—Test your answer, giving values to the letters. Also inspect the figure.

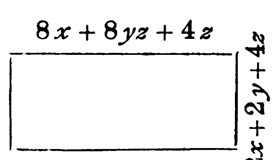


FIG. A.

277. What is the perimeter of Fig. A? From the perimeter take the sum of the vertical sides. What remains?

278. Take the sum of the two horizontal sides from the perimeter and what remains?

NOTE.—Write the work out in full. Inspect your work and see whether your answer is correct.

279. From one of the horizontal sides take one of the vertical sides, and then take this difference from the whole perimeter. What remains?

NOTE.—For explanation of the process for polynomials see page 110. These questions may be omitted at the discretion of the teacher.

(*a*). From $3x + 2yz + 16yz$ take $-2x + 2yz - 8xyz$.

(*b*). Take from $(18xyz + 16z - 19yz) - 9xyz + 4z - 19yz$.

(*c*). Find the difference between $25x^3y + 2xy^3 - 14xyz^2$, and $25x^2y + 2xy^3 - 14xyz^2$.

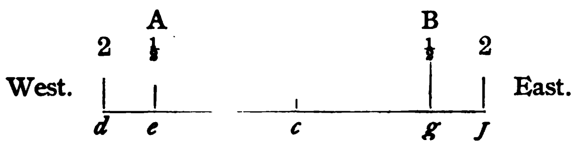
(*d*). If from $2z + 2x - 2y + xy$ we take $2z + 2x - 2y + xy + az$ what will remain?

(*e*). $27abc + 18ab - a$; $+6abc - 18ab + 28a$. Find the difference.

(*f*). $2xy$ taken from $xy + 6x - 3z$ equals what?

(*g*). Take $xy + 6x - 3z$ from $2xy$ and what remains?

280. From $x - y$ take $-x + y$, and what remains? Illustrate by means of a diagram.



NOTE.—The problem might read as follows: Two men, A and B, started from the same place on Monday morning. On Monday A walked x miles west and on Tuesday y miles east. On Monday B walked x miles east and on Tuesday y miles west. How far is A from B?

(*a*). Let it be understood that west is positive, then east must be negative.

(*b*). The answer is the difference between $x - y$ and $-x + y$, or $2x - 2y$.

(*c*). Proof: Let $+x = 2$ miles and $y = \frac{1}{2}$ mile. Then see diagram drawn on a scale of $\frac{1}{2}$ inch to a mile. c represents the starting point. Then to show how far A walked on Monday we must draw

a line 1 inch to the left of c to d. On Tuesday he walked *east* y miles; hence, $-y$ or $\frac{1}{2}$ mile east, see point e $\frac{1}{4}$ inch from d.

On Monday B walked x miles *east*; hence $-x$ miles or 2 miles east, see point f 1 inch from c. On Tuesday he walked y miles *west*, hence $+y$ or $\frac{1}{2}$ mile west; see point g $\frac{1}{4}$ inch from f.

Giving values to x and y we have $+4-1$.

Simplifying the answer, which in algebra means to unite the terms of a polynomial or equation, we have $+3$, which means A is 3 miles from B west, shown by point e, $1\frac{1}{2}$ inches from g. By this you see that although we added the quantities we really got the difference.

(*i*). From $x+y$ take $-x-y$ and show with a diagram that your answer is right.

(*j*). Take $-x-y$ from $-x+y$ and show your answer is correct.

LESSON XXIX.

281. How long is the side 4 5? (Read four-five.)

282. What does $z-y$ represent here?

283. Calling the length of the side 4 5, then, $z-y$, does $z-z-y$ give the side 2 3? Why?

284. When I take z from z how much is left? What must I do to get 2 3?

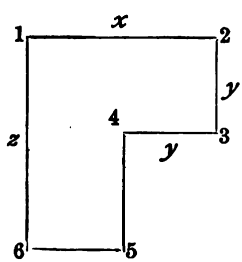


FIG. 10.

285. What does $z-(z-y)$ represent here?

286. Perform the operation indicated in example 285.

287. What is the difference between $z-(z-y)$ and $z-z+y$?

288. What are the signs of z and y when in the parenthesis?

289. What are the signs of z and y when the parenthesis is removed?

290. Copy or read and fill in the blanks. If two or more terms are inclosed in a parenthesis ***preceded by minus***, the______, and the sign before it may be removed, if the sign of every term inclosed is______.

Remove the parenthesis from the following:

291.	292.	293.
$2 + (y - z)$	$5x - (a + b)$	$7y - (x + 2y)$
294.	**295.**	**296.**
$7y - (x - 2y)$	$7y + (x - 2y)$	$3x - (6y + z)$

Inclose the last two terms of the following trinomials in parenthesis:

297.	298.	299.	300.
$x + y - z$	$x - y + z$	$x - y - z$	$2x - y - 2z$

290*a*. Copy or read and fill the blanks: Any number of______may be inclosed by a______with a minus sign before it; if the sign of every term inclosed be______.

290*b*. Copy or read and fill the blanks: If a number of terms are inclosed by a______with a plus sign before it, the______and the sign______it may be removed without altering the value of the expression.

Write answers to the following in two ways: (1) using the parenthesis; (2) without using the parenthesis:

(*a*). From x take the sum of y and z.
(*b*). From z take the difference between x and y.
(*c*). Add the sum of x and y to z.
(*d*). Add to z the difference between x and y.

NOTE.—See Problems, Exercise VII, page 104 for applications of the above. It will be well to work them before beginning the next subject

LESSON XXX.

MULTIPLICATION.

(*a*). In arithmetic the multiplier tells *how many* equal numbers are to be taken.

(*b*). In algebra the multiplier tells how many equal numbers, or quantities, are to be taken; and it also tells *how the product is to be taken, i.e.*, whether *additively* or *subtractively*.

(*c*). When the product is to be taken *additively* the multiplier has a + sign.

(*d*). When the product is to be taken *subtractively* the multiplier has a − sign. Remember this.

Read carefully and answer the following questions:

(*a*).	(*b*).	(*c*).	(*d*).
$+4x$	$-4x$	$+4x$	$-4x$
$+3$	$+3$	-3	-3

NOTE.—See page 111.

I. In example (*a*) how many times is $+4x$ to be taken? What will $+4x+4x+4x$, or 3 times $+4x$ equal? *How* is the product to be taken in example (*a*)? $+12x$ taken additively equals what? Then what is the sign of the product in example (*a*)?

II. In example (*b*) how many times is $-4x$ to be taken? What will $-4x-4x-4x$, or 3 times $-4x$, equal? *How* is the product to be taken in example (*b*)? $-12x$ taken additively equals what? Then what is the sign of the product in example (*b*)?

III. In example (*c*) how many times is $+4x$ to be taken? What will $+4x+4x+4x$, or 3 times $+4x$, equal? *How* is the product to be taken in example (*c*)? Why do you say subtractively? Taking the product $+12x$ sub-

tractively makes it what? If taking $+12x$ subtractively makes it a *subtrahend*, what must we do to the sign? Then what is the sign of the product in example (*c*)?

IV. In example (*d*) how many times is $-4x$ to be taken? What will $-4x-4x-4x$, or 3 times $-4x$, equal? *How* is the product to be taken in example (*d*)? Why subtractively? Taking the product $-12x$ subtractively makes it what? If taking $-12x$ subtractively makes it a *subtrahend*, what must we do to the sign? What, then, is the sign of the product in example (*d*)?

(*e*).	(*f*).	(*g*).	(*h*).
$+4x$	$-4x$	$+4x$	$-4x$
$+3$	$+3$	-3	-3
$+12x$	$-12x$	$-12x$	$+12x$

Inspect the examples (*e*), (*f*), (*g*) and (*h*), and tell *when* the sign of the product is + and when it is −.

300*a*. Copy or read and fill in the blanks. In algebraic multiplication *like* signs give ——— and unlike signs give ———.

LESSON XXXI.

Name at sight the products in the following:

301.	302.	303.	304.	305.
$-5x$	$+8x$	$+7x$	$-9x$	$+6x$
$+4$	$+8$	-6	-7	-5

306.	307.	308.	309.	310.
$-12y$	$4y$	$-3y$	$2y$	$-9y$
8	-9	-7	4	-9

311.	312.	313.
$12x + 3y$	$4x - 6y$	$+9x - 8y$
-3	4	-7

314.	315.	316.	317.
xy	xyz	$-x + y$	$x - y$
x	$-x$	$-x$	x

LESSON XXXII.

318. What is the perimeter of Fig. 11?

319. Represent $\frac{1}{2}$ of its perimeter?

320. What is the area of Fig. 11? Write the answer in two ways.

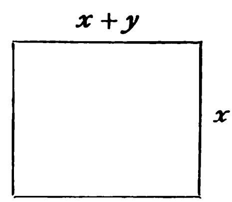


FIG. 11.

NOTE.—Give values to x and y, as x = 13 inches and y = 3 inches. Then the area is 13 times 16 sq. in. = 208 sq. in. See if $x^2 + xy$ equals 208 sq. in.; also $x(x + y)$. Test, the perimeter also = 58 inches. Does $2x + 2(x + y)$ equal 58 inches? Does $(x + y) + x + (x + y) + x$?

321. What is the perimeter of Fig. 12?

322. What is $\frac{1}{2}$ of its perimeter?

323. What is $\frac{1}{4}$ of its perimeter?

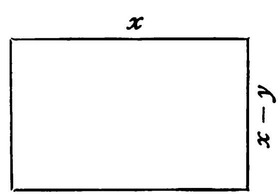


FIG. 12.

324. What is the area of Fig. 12?

NOTE.—Let $x = 20$ and $y = 7$, and test as before.

LESSON XXXIII.

325. What is the perimeter of Fig. 13? Write the answer in four ways.

326. What is $\frac{1}{4}$ of the perimeter?

327. What is $\frac{1}{8}$ of the perimeter?

328. What does $\frac{x+y}{4}$ represent?

329. What is the area of Fig. 13?

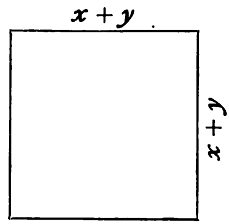


FIG. 13.

NOTE.—Let $x = 6$ and $y = 4$. The area is then 100. Test your answer $x^2 + 2xy + y^2$ and see whether it is correct.

330. What does $(x+y)^2$ represent? See Fig. 13.

331. What is the area of a square the length of whose sides is $a + b$?

332. What is the area if the sides were $y + z$ long?

333. What is the area if the length of the sides were $2x + 2y$? If $2a + 2b$?

LESSON XXXIV.

THEOREMS.

Name at sight the answer to the following:

334.	335.	336.	337.
$(x+y)^2$	$(y+z)^2$	$(a+b)^2$	$(b+c)^2$
338.	339.	340.	341.
$(2x+2y)^2$	$(2a+2b)^2$	$(2c+2d)^2$	$(3x+2y)^2$

342.	343.	344.
$(4x+3y)^2$	$(5x+5y)^2$	$(ab+bc)^2$

345. Copy or read and fill the blanks: The square of the sum of two quantities is equal to the square of the ——, plus twice the product of the —— by the —— plus the square of the ——.

This is known in algebra as *Theorem I.*

LESSON XXXV.

346. What is the perimeter of Fig. 14?

347. What is $\frac{1}{3}$ the perimeter?

348. Represent $\frac{1}{3}$ of $\frac{3}{4}$ of the perimeter.

349. What is the area of Fig. 14?

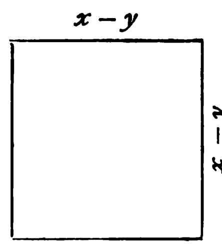


FIG. 14.

NOTE.—Give values to the letters and see whether your answer $x^2 - 2xy + y^2$ is correct. $x = 4$, $y = 2$.

350. What is the area of a square the length of whose sides is $a - b$?

351. What if the length of the sides of a square is $2x - 2y$?

352. What does $(x - y)^2$ represent?

Name at sight the answers to the following:

353. $(x - y)^2$	354. $(2x - 2y)^2$	355. $(2a - 2b)^2$	356. $(2b - 2c)^2$
357. $(5x - 5y)^2$	358. $(4y - 4z)^2$	359. $(3a - 3b)^2$	360. $(6z - 6y)^2$

360*a*. Copy or read and fill the blanks: The square of the difference of two quantities is equal to the square of the______minus twice the product of the______by the ______plus the square of the______.

This is known in algebra as *Theorem II.* We shall often use the theorems. They will save us much time if we only remember them.

LESSON XXXVI.

361. Find the perimeter of Fig. 15.

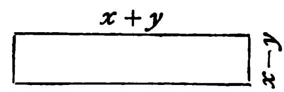


FIG. 15

362. From the perimeter take the sum of the two ends. What remains?

363. Take the sum of one end and one horizontal side from the perimeter and tell what remains?

364. How long will the perimeter be if the sum of the two longer sides is taken from it?

365. What is the area of Fig. 15?

NOTE.—Let $x = 6$ and $y = 4$ and test your answers.

366. What does $x^2 - y^2$ represent? See Fig. 15.

367. What is the area of a rectangle whose sides are $a + b$ by $a - b$?

368. What is the area of a rectangle whose sides are $2x + 2y$ by $2x - 2y$?

369. What does $(x + y)(x - y)$ represent?

Name at sight the answers to the following:

370.	371.	372.
$(y+z)(y-z)$	$(2y+2z)(2y-2z)$	$(4a+2b)(4a-2b)$

373.	374.
$(4x + 2y)(4x - 2y)$	$(5x + 5z)(5x - 5z)$
375.	**376.**
$(6a + 4b)(6a - 4b)$	$(7x + 3y)(7x - 3y)$

376*a*. Copy or read and fill the blanks. The *product* of the sum and difference of two quantities is equal to the ______ of their ______.

This is known in algebra as *Theorem III.*

LESSON XXXVII.

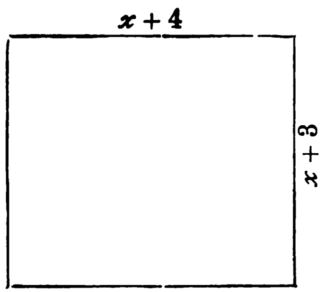


FIG. 16.

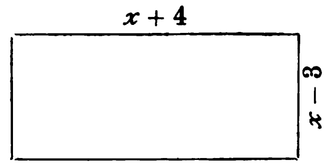


FIG. 17.

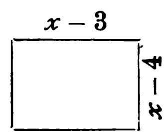


FIG. 18.

377. From the perimeter of Fig. 16 take one end. What remains?

378. From the perimeter of Fig. 17 take one of the longer sides. What remains?

379. Take one of the shorter sides of Fig. 18 from its perimeter. What remains?

380. From one of the longer sides of Fig. 17 take one end, and then take the difference from its perimeter.

NOTE.—Always write your work out in full. Test your answers, letting $x = 8$.

381. Find the area of Fig. 16.
382. Find the area of Fig. 17.
383. Find the area of Fig. 18.

NOTE.—We will now compare the answers to the last three problems.

$$\text{Fig. } 16 = x^2 + 7x + 12.$$
$$\text{Fig. } 17 = x^2 + x - 12.$$
$$\text{Fig. } 18 = x^2 - 7x + 12.$$

384. The length of Fig. 16 is the binomial $x + 4$. Its width is the binomial $x + 3$. What term is common to both binomials?

385. Square the common term, and what do you obtain?

386. How does this compare with the first term of your product?

387. What is the algebraic sum of the unlike terms?

388. Multiply this sum by the common term. What is the result? How does it compare with the second term of your product?

389. Multiply your unlike terms together. What is the result? How does it compare with the third term of your product?

NOTE.—Test the next two in the same manner.

389*a*. Copy or read and fill in the blanks:

The product of two binomials having a common term equals the ______ of the ______ term and the product of the common term by the algebraic ______ of the ______ terms and the algebraic ______ of the unlike terms. Remember this; it will often be useful.

LESSON XXXVIII.

390. What is the area of a rectangle $(x + 5)$ by $(x + 2)$?

391. What is the area of a rectangle $(x + 6)$ by $(x - 5)$?

392. What is the area of a rectangle $(x - 7)$ by $(x - 10)$?

393. What is the area of a rectangle $(a + b)$ by $(a + 4b)$?

394.	395.
$(y + 5)(y + 10) = ?$	$(y + z)(y - 3) = ?$
396.	397.
$(z + 2)(z - 4) = ?$	$(a + b)(a + c) = ?$
398.	399.
$(2z + 3)(2z - 7) = ?$	$(a - 3)(a - 7) = ?$

400. Find the area of Fig. 19. Of Fig. 20.

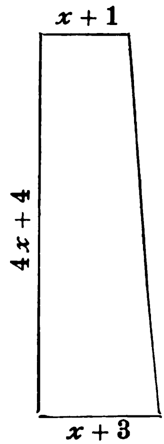


FIG. 19.

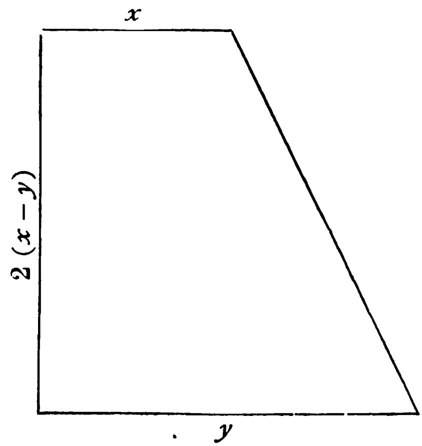


FIG. 20.

LESSON XXXIX.

PROBLEMS.

401. What have we learned about multiplying both members of an equation by the same or equal quantities?

402. Multiply both members of the equation $-4x = -16 - 4$ by -1, and read your answer.

403. To what number can 2 be added, making $\frac{1}{4}$ the sum equal to $\frac{1}{3}$ the number?

404. Robert had a certain number of dollars, to which on his birthday his father added \$8. He then spent $\frac{1}{3}$ of his money for a set of Dickens, which cost him $\frac{1}{2}$ of what he had at first. What did the books cost?

405. There is a number which, if we multiply by 3 and take 12 from the product, $\frac{1}{8}$ the difference will equal the difference between $\frac{1}{4}$ of 6 times the same number and 24. Find the number.

406. $\frac{1}{5}$ of a given number added to 3 equals $\frac{3}{10}$ of itself diminished by the fraction $\frac{2}{5}$. Find the number.

407. Seven times a certain number plus 3 equals 9 times the same number less 9. Find the number.

408. John has a certain number of dollars; $\frac{5}{6}$ of his money equals $\frac{3}{4}$ of his money added to $20. How much money has he?

409. The sum of two numbers is 96. If the greater is divided by the less the quotient is 5. Find the numbers.

410. The difference between two numbers is 24. The greater divided by the less equals 3. Find the numbers.

411. If William is 3 times as old as Henry now, and in 10 years will be twice as old, how old is he? How old is Henry?

412. A can do a piece of work in 4 days. B can do the same work in 6 days. If they work together, how long will it take them?

413. How long will it take 3 men working together to do a piece of work, if A can do it in 3 days, B in 2 days and C in $\frac{1}{2}$ day?

414. If A can dig a well in 5 days, B in 4 days and C in $3\frac{1}{3}$ days, how long will it take them to dig it working together?

NOTE.—Fill the blanks. If the sign of each term of an equation is changed the——is not destroyed. Changing the signs thus is equivalent to——both members of the equation by——one.

LESSON XL.

DIVISION.

NOTE.—See page 112 for model example in division of Polynomials.

Algebraic division is the process of finding how many algebraic quantities, ***and in what manner***, one of them must be taken to produce the other:

(*a*).	(*b*).	(*c*).	(*d*).
$\frac{8x^3}{2x}=4x^2$	$\frac{8x^3}{-2x}=-4x^2$	$\frac{-8x^3}{2x}=-4x^2$	$\frac{-8x^3}{-2x}=4x^2.$

I. Look at Examples (*a*), (*b*), (*c*) and (*d*). What is the dividend? What do we call $2x$?

II. In Example (*a*) what quantity do we wish to produce? Is it a positive or a negative quantity?

III. What quantity have we given? Is it positive or negative? Then *how* must we take it to produce $+8x^3$?

IV. How many must we take to produce $+8x^3$? Then the quotient is what?

V. What kind of quantity is the divisor in Example (*b*)? What kind of quantity do we wish to produce? How must we take a negative quantity to produce a positive one? Then what is the quotient?

VI. What kind of quantity is the divisor in Example (*c*)? What kind of quantity do we wish to produce? How must we take a positive quantity to produce a negative quantity? Then what is the quotient?

VII. What kind of quantity is the divisor in Example (*d*)? What kind of quantity do we wish to produce? How must we take a negative quantity to produce a negative quantity? Then what is the quotient?

414*a*. Copy or read and fill the blanks. In division like signs give______, and unlike signs give______.

Name at sight the quotients in the following examples:

415. $\dfrac{6x^3}{3x^2}$

416. $\dfrac{12x^2y}{-4x^2}$

417. $\dfrac{-9x^2y^2}{-3y^2}$

418. $\dfrac{15x^3y}{-3x^2y}$

419. $\dfrac{42x^3y^2z}{7xyz}$

420. $\dfrac{-18x^3y^3}{6x^3y}$

421. $\dfrac{22x^3}{11}$

422. $\begin{array}{r|l} -48x^3y^2 & \underline{6x^2} \\ & \end{array}$

423. $\begin{array}{r|l} 49y^3z^3 & \underline{7z^3} \\ & \end{array}$

424. $\begin{array}{r|l} 144x^3y^3z^3 & \underline{-12x^2y^2z^2} \\ & \end{array}$

425. $\begin{array}{r|l} 64abc^3 & \underline{8c^3} \\ & \end{array}$

NOTE.—In the following examples do not forget your theorems:

426. $\dfrac{x^2+2xy+y^2}{x+y}$

427. $\dfrac{x^2-2xy+y^2}{x-y}$

428. $\dfrac{x^2+7x+12}{x+4}$

429. $\dfrac{x^2+x-12}{x-3}$

430. $\dfrac{x^2-y^2}{x-y}$

431. $\dfrac{x^2-y^2}{x+y}$

NOTE.—Work an original example in multiplication; then prove it by division, and *vice versa*. Hundreds of tests may thus be applied in multiplication and division.

LESSON XLI.

432. How long is the vertical side of Fig 21?

$$6x^2 + 5xy + y^2 \,|\, \underline{3x + y}$$

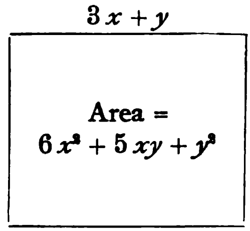


FIG. 21.

433. If the area of a rectangle is $2x^2 + 5xy + 3y^2$, and one side equals $x + y$, what must equal the other side?

434. Find the length of a rectangle whose area is equal to $x^3 + y^3$, and whose width equals $x + y$.

435. The area of a trapezoid is $4x^2 + 2xy - 2y^2$, the sum of its parallel sides $2x + 2y$. What is its altitude?

436. The base of a triangle equals $2x + y$; its area equals $8x^3 + 8xy + 2y^2$. What is its altitude?

437. The altitude of a trapezoid is $6x - 6y$; its area is $6x^2 + 3xy - 9y^2$. What is the sum of its parallel sides?

438. What must be the length of the altitude of a triangle whose base is $x + 2y + z$, and whose area is $3x^2 + 6xy + 3xz$?

439. The area of a circle is $9x^2 + 9xy + 9xz + 9yz$; $\frac{1}{2}$ its diameter is $3x + 3y$. What is its circumference?

440. The area of a square is $25x^2 + 50xy + 25y^2$; one side equals $5x + 5y$. What must the other side equal? What if the area had been $25x^2 - 50xy + 25y^2$?

441. What must be the perimeter of a room that is y feet high, the area of whose four walls is $2y^2 + 4yz$ square feet?

442. $x^4 - y^4 \div x - y = ?$

443. $x^5 + 32y^5 \div x + 2y = ?$

444. $x^4 + x^2y^2 + y^4 \div x^2 - xy + y^2 = ?$

NOTE.—Other problems may be obtained by multiplying two quantities, as $3x + 2y + y$ by $3x + y + z$, and then proving the work by division.

LESSON XLII.

EQUATIONS CONTAINING TWO UNKNOWN QUANTITIES.

445. What have we learned about adding the same or equal quantities to both members of an equation?

446. The sum of two numbers is 12 and their difference is 4. What are the two numbers?

Let x = the larger number,
and y = the smaller number.
Then will (1) $x + y = 12$,
and (2) $x - y = 4$.

447. Which is the greater in equation (2), $x - y$ or 4?

448. Then, if we add $x - y$ to the first member of equation (1) and add 4 to the second member of equation (1), what shall we have done?

Let us do this:

$$\begin{array}{ll} (1) & x + y = 12 \\ (2) & x - y = 4 \\ \hline (3) & 2x \quad = 8 \\ (4) & \ \ x \quad = 4 \end{array}$$

449. After adding the members of the second equation to the members of the first equation, what do we get for the third equation?

450. Substituting the value of x in the first equation we have $4 + y = 12$

Transposing 4 we have $y = 12 \dot{-} 4$

$y = 8$

451. 2 times Gordon's marbles added to 2 times Charlie's equals 10, or 4 times Gordon's added to 2 times Charlie's equals 16. How many marbles has each boy?

Let x = the number Gordon had, and
y = the number Charlie had.

$$\begin{array}{rrrl}
 & (1) & 2x + 2y = & 10 \\
 & (2) & 4x + 2y = & 16 \\
\hline
\textit{Subtracting } (2) \text{ from } & (1) & -2x \qquad = & -6 \\
 & & x \qquad = & 3 \\
\text{Substituting} & (1) & 6 \quad + 2y = & 10 \\
 & & 2y = & 10 - 6 \\
 & & y = & 2
\end{array}$$

452. Performing such operations upon two given equations as are necessary to get rid of one of the unknown quantities is called *Elimination.*

(a). In the first problem were the coefficients of y equal or unequal? Were its signs like or unlike? What did we do to eliminate y?

(b). In the second problem were the coefficients equal or unequal? Were its signs like or unlike? What did we do to eliminate y?

453. Copy or read and fill the blanks: When a quantity is to be eliminated its coefficients must be______, and if its signs are______we add one member to the other. If its signs are______we subtract one member from the other.

$$(1)\ 2x + y = 13$$
$$(2)\ x + 2y = 11$$

454. Is there any way to make the coefficients of y equal in the above equations?

$$(3)\ 4x + 2y = 26$$
$$(2)\ x + 2y = 11$$

455. What was done to (1) to make equation (3)? Find the values of x and y.

$$(1)\ 2x + 2y = 20$$
$$(2)\ x + y = 10$$
$$(3)\ x + y = 10$$

456. In this case what was done to (1) to get (3)?

LESSON XLIII.

457. The sum of A's and B's ages is 35 years, and if twice B's age be taken from twice A's age the difference will equal 10. How old is each?

458. Mary bought 2 slate pencils and 3 lead pencils at cost of 21 cents. Had she bought 5 slate pencils and 4 lead pencils the cost would have been 35 cents. What were the pencils worth apiece?

459. The sum of two numbers is 15. The greater divided by the less equals 2. Find the numbers.

460. Ten years ago a father was 7 times as old as his son. Now he is only three times as old. Find the age of each.

461. George and Henry spent the afternoon collecting shells and then sat down to count them, when it was found that they had 60 between them; but George had 4 times as many as Henry. How many shells had each boy collected?

462. Two numbers whose difference is 68 are to each other as 15 to 3. Find the numbers.

463. There are two numbers to whose sum if 14 be added the sum will be 100; but if the less be taken from the greater the difference will equal 32. Find the numbers.

464. "There is a fraction such, that if you add 1 to its numerator, it becomes $\frac{1}{2}$; but if you add 3 to its denominator it becomes $\frac{1}{3}$. Required the fraction."

465. There is another fraction which if 1 be added to its numerator, it becomes 1; but if 2 be added to its denominator it becomes $\frac{1}{2}$. What is the fraction?

466. Two numbers are to each other as 4 is to 2, and $\frac{1}{2}$ the greater added to $\frac{1}{3}$ the less equals 8. What is the sum of the numbers?

467. A fish line is $\frac{1}{6}$ of its length in the water; 6 feet of the line is wound on the end of the pole; the rest of the line is $\frac{3}{4}$ of its whole length. How long is the line?

LESSON XLIV.

468. There are two pipes used to fill a water tank One of them will fill it in an hour and a half; the other requires $2\frac{1}{4}$ hours. In what time will both fill it?

469. If the tail of a fish weighs 9 lbs., his head as much as his tail and half his body, and his body as much as his head and tail, what is the weight of the whole fish?

470. A and B sell papers. The first day A gains as much money as he had at first and $4 more, and finds he has twice as much money as B. The second day B gains half as much as he had at first and $1 more, when he finds he has $\frac{7}{8}$ as much as A. How much money had each at first?

471. If John gives Willie 20 marbles he will still have twice as many as Willie; but if he gives him 40 they will each have the same number. How many has each?

472. James bought 20 oranges and 30 lemons, for which he paid $1.70. Had he bought 30 oranges and 20 lemons they would have cost him $1.80. What did the oranges and lemons cost apiece?

473. If A gives $10 of his money to B, they will each have the same amount of money. If B gives $10 to A, B's money will equal but $\frac{3}{5}$ of A's original sum. Find how much money each man has.

474. If Henry gives $10 of his money to Frank, they will have equal sums; but if Frank gives $10 to Henry, Frank's money will equal only $\frac{9}{29}$ of Henry's. How much more had Henry than Frank?

475. The difference between two numbers equals $1\frac{1}{3}$ times the less number; their sum equals 12 more than the larger number. Find the numbers.

476. Three times W's age added to twice C's age equals 165 years, and $\frac{1}{5}$ of W's age added to C's age equals 24 years. How much older is W than C?

477. The carpet and furniture in a parlor are together worth \$1500; the furniture is worth 5 times as much as the carpet. What is the value of each?

478. Two fractions are to each other as 1 is to 2; their sum is one. Find the value of each.

LESSON XLV.

479. There are two numbers. If 33 be added to 3 times the second, the sum will equal 8 times the first; or 2 times the second plus 20 equals 5 times the first. What are the numbers?

480. There are two other numbers. If 3 times the first be divided by 5 times the second, the quotient will be $2\frac{2}{5}$; or, if 1 be added to the first $\frac{1}{9}$ the sum will equal the difference between $\frac{1}{3}$ of 2 times the second and $\frac{1}{3}$. Find the numbers.

481. There are two numbers. $\frac{2}{3}$ of the smaller number, added to $\frac{3}{4}$ of the larger one, equals 26. Or, $\frac{3}{4}$ of the smaller, added to $\frac{2}{3}$ of the larger, equals 25. What are the numbers?

482. One-fifth of the sum of two numbers equals 1 less than one dozen, and their difference equals 11. What is the numerical sum of the two numbers?

482*a*. If to 5 times the sum of two numbers 20 be added, the sum will be three score and ten. These numbers are to each other as 12 is to 8. Required the numerical difference between them.

LESSON XLVI.

FACTORING.

(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*f*)
3		x		x	
4	12	x	x^2	y	xy

(1). What was done to the numbers in example (*a*) to get the number in example (*b*)? (2). What was done to the quantities in example (*c*) to get the quantity in example (*d*)? (3). What was done with the quantities in (*e*) to get the quantity in (*f*)? What are the factors of 12? (4). What are the factors of x^2? Of xy? Copy or read and fill the blanks:

483. The quantities multiplied together to produce a ______are the______of the quantity.

What are the FACTORS of the following quantities?

(1).	(2).	(3).	(4).
xy	yz	xyz	x^2y^2

NOTE.—The operations of factoring are performed by inspection. Divide xy, Ex. (1), by x. What is the quotient? Divide yz, Ex. (2), by y. What is the quotient?

I. What are the factors of xy? (Ex. 1.)

II. Divide xy by x.

III. What is the quotient? The divisor?

IV. How do the quotient and divisor compare with the factors of xy?

Fill the blanks: A divisor of a quantity is one of the factors of the quantity, and the______is the other______.

NOTE.—The operations of factoring are performed by inspection. A monomial that is a common factor to all the terms of a polynomial may be placed without a parenthesis, and the other terms within.

Find the factors of the following:

1. $x^2y^2 + x^2z^2 + x^2b = x^2\ (y^2 + z^2 + b)$.
2. $6\,x^3y - 12\,x^2y^2 + y^2z = y\ (6\,x^3 - 12\,x^2y + yz.)$
3. $a^2b^2 - ab^2 + a^2b$.
4. $y^3 + 3\,y^2 - 2\,y$.
5. $3\,y^2 - 6\,yz + 9\,yz^2$.

LESSON XLVII.

NOTE.—Remember (1) that the factors of a quantity are such quantities as multiplied together will produce the quantity. (2) To apply what you have learned. There is nothing new required of you in this lesson. Think a moment and you will have no trouble with Examples 488 or 490. (3) Remember always to use a theorem when you can.

Factor the following quantities:

484. x^2.
485. xy.
486. $x^2 - y^2$.
487. $z^2 - y^2$.
488. $x^2 + 2\,xy + y^2$.
489. $x^2 - 2\,xy + y^2$. .
490. $x^2 + 8\,x + 16$.
491. $9\,x^2 - 6\,xy + y^2$.
492. $12\,x^4yz^3 - 15\,x^3z^4 - 6\,x^2z^3b$.

LESSON XLVIII.

What are the factors of each of the following quantities? Think a moment and you will have no trouble with Ex. 497 and others like it:

493. z^2.
494. xyz.
495. $a^2 - b^2$.
496. $4\,a^2 - 9\,b^2$.
497. $a^2 - 8\,a + 15$.
498. $x^2 + 7\,x + 12$.
499. $x^2 + x - 12$.
500. $x^2 - 7\,x + 12$.
501. $6\,xy^2 - 12\,x^2y^3 - 36\,xy$.
502. $144\,a^2 + 264\,ab + 132\,b^2$.

LESSON XLIX.

NOTE.—See Lesson IX., Note to question 67.

(*a*).	(*b*).
$x^2 - y^2$	$x^2 - (y + z)^2$

I. How many terms in the quantity under (*a*)?

II. How many terms in the quantity under (*b*)?

III. Why only two terms in the quantity under (*b*)?

IV. What is the first term in (*b*)? The second term?

V. What factors produced the quantity in (*a*)?

503. What are $(x - y)$ and $(x + y)$ with reference to Ex. (*a*)?

504. What are $x + (y + z)$ and $x - (y + z)$ with reference to Ex. (*b*)?

505. Remove the parentheses from the second terms in 504. (See Lesson XXIX, 290.)

506. Factor $a^2 - (b + c)^2$.

507. Factor $4x^2 - (x + y)^2$.

LESSON L.

Find the factors of the following quantities. There is nothing new required. Remember that we factor by inspection. Ex. 513 and others like it are nothing new:

508. $x^2 + xy$.

509. $x^4 - y^4$.

510. $z^2 - (x + y)^2$.

511. $(x + y)^2 - z^2$.

512. $x^2y^2 - y^2z^2$.

513. $x^2 + (a + b)x + ab$.

514. $x^2 - (a + b)x + ab$.

515. $x^2 - (y + z)x + yz$.

516. $(x + y)^2 - (x - y)^2$.

517. $12a^4bc^3 - 15a^3c^4 - 6a^2c^3d$.

LESSON LI.

518. Mr. A owns a lot $x + y$ rods long and $x + y$ rods wide. What is the shape of the lot?

519. What is the area of Mr. A's lot? (See Problem 518.)

520. Mr. B owns a lot whose area is 5 times as great as A's. What is the area of Mr. B's lot?

521. Factor $5x^2 + 10xy + 5y^2$.

522. Factor $9a^2 + 18ab + 9b^2$.

523. Factor $8y^2 + 24yz + 18z^2$

524. Mr. C owns a lot that is $x + 3$ rods long and $x + 2$ rods wide. What is the shape of the lot?

525. What is the area of Mr. C's lot?

526. Mr. D owns a lot whose area is twice as great as Mr. C's. What is the area of Mr. D's lot?

527. Factor $2x^2 + 10x + 12$.

528. Factor $5x^2 - 5x - 30$.

LESSON LII.

Factor the following:

529. $8x^2 + 40x + 48$.

530. $7x^2 - 7x - 42$.

531. $6x^2 - 12xy + 6y^2$.

532. $5x^2 - 5y^2$.

533. $2x^2 - 20x + 48$.

534. $14a^3b^2c + 7a^2b^3c - 21a^2bd$.

535. $5x^2 - 5(a + b)^2$.

536. $16a^2 + 32ab + 16b^2$.

LESSON LIII.

537. William has a block of wood $x+y$ inches long, $x+y$ inches thick and $x+y$ inches wide. What is the shape of the block?

538. What is the volume of William's block of wood?

539. If $x = 1$ and $y = 2$, what would be the volume of William's block? Test your answer and see whether it is correct.

540. Read and complete the sentences below after careful inspection of the answer to Problem 538:

The cube of the sum of two quantities equals the ______of the first, plus______times______of the first by the second, plus __ times the______by the______of the second, plus the______of the second.

NOTE.—Performing the multiplication indicated is called *expanding the expression.*

541. Expand $(a+b)^3$.
542. Expand $(y+2)^3$.
543. Expand $(2+z)^3$.
544. Expand $(x-y)^3$.
545. To what is the cube of the difference of two quantities equal?

NOTE.—You should be able to write this unaided.

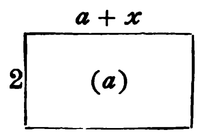
546. Expand $(a-b)^3$.
547. Expand $(2x+xy)^3$.

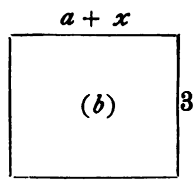
LESSON LIV.

548. Factor $x^3 + 3x^2y + 3xy^2 + y^3$.
549. Factor $x^3 - 3x^2y + 3xy^2 - y^3$.
550. Factor $8 + 12z + 6z^2 + z^3$.
551. Factor $8x^6 - 36x^4y^2 + 54x^2y^4 - 27y^6$.

552. Factor $2x^3 + 6x^2y + 6xy^2 + 2y^3$.
553. Factor $4x^3 - 12x^2y + 12xy^2 - 4y^3$.
554. Factor $8x^3 - 24x^2y + 24xy^2 - 8y^3$.
555. Factor $6x^3 + 18x^2y + 18xy^2 + 6y^3$.

LESSON LV.

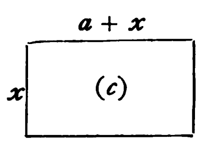


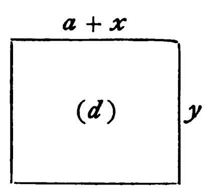


556. What is the area of rectangle (*a*) expressed in the shortest way?

557. What is the area of rectangle (*b*) expressed in the shortest way?

I. Since rectangle $(a) = 2(a + x)$ and rectangle $(b) = 3(a + x)$, what is the sum of the two areas? $5(a + x)$ we see at once is the answer.





558. What is the area of rectangle (*c*) expressed in the shortest way?

559. What is the area of rectangle (*d*) expressed in the shortest way?

II. Since rectangle $(c) = x(a + x)$ and rectangle $(d) = y(a + x)$, what is the sum of the two areas? $(x + y)(a + x)$ we see at once is the answer.

560. Factor the polynomial $2(a + x) + 3(a + x)$.
561. Factor the polynomial $x(a + x) + y(a + x)$.
562. Factor the polynomial $2a + 2x + 3a + 3x$.
563. Factor the polynomial $ax + x^2 + ay + xy$.

NOTE.—We see by inspection that the above polynomials, Examples 562, 563, consist of four terms, of which the first two and the last two contain a common binomial factor. All such polynomials are factored as shown above.

Factor the following:

564. $a^2 + ax + ab + bx$.
565. $xy - xz + ay - az$.
566. $x^2y + x^2z - 4y - 4z$.
567. $ab - bc - ad + dc$.
568. $a^2x - a^2 - x + 1$.
569. $ax^2 - ay^2 + bx^2 - by^2$.
570. $5ab - 5bc + 3a^2 - 3ac$.
571. $a^2b^2 - a^2c^2 + b^2x^2 - c^2x^2$.

REVIEW EXERCISE.

1. What are the factors of $25 + 20x + 4x^2$?
2. Write the factors of $9x^2 - 30x + 25$.
3. What are the factors of $9x^2 - 6xy + y^2$?
4. Name the factors of $4x^2 - 12xy + 9y^2$.
5. What are the factors of $8 + 12y + 6y^2 + y^3$?
6. Write the factors of $25x^2 + 35x + 6$.
7. What are the factors of $z^2 + 11z + 18$?
8. What are the factors of $4x^2 - 4y^2$?
9. Factor $x^2 - (y + z)^2$.
10. What are the factors of $a^2b^2 - b^2c^2$? Tell which theorem was used in each example.

LESSON LVI.

FRACTIONS—REDUCTION.

The first thing for us to know when beginning the study of fractions is that the reasoning is the same, whether the fractions be expressed by figures or by letters.

The same signs that we have had in arithmetic are to be used, as ×, −, +, ÷; also the same names, as numerator, denominator, least common denominator, etc.

(*a*).	(*b*).	(*c*).	(*d*).
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{6}{8}$
(*e*).	(*f*).	(*g*).	(*h*).
$\frac{x}{y}$	$\frac{x^2}{xy}$	$\frac{y^2}{xy}$	$\frac{y}{x}$

I. What was done in Example (*a*) with the numerator and denominator to get the fraction in Example (*b*)?

II. What was done in Example (*g*) with both terms of the fraction, *i.e.*, numerator and denominator, to get the fraction in Example (*h*)?

III. What was done in Ex. (*c*) to get Ex. (*d*)?

IV. What was done in Ex. (*e*) to get Ex. (*f*)?

V. How does the fraction in (*a*) compare with the fraction in (*b*)?

VI. How does the fraction in (*g*) compare with the fraction in (*h*)?

NOTE.—If in doubt, give values to x and y, as $x = 6$, $y = 5$, and test.

VII. How does the fraction in (*c*) compare with the fraction in (*d*)? How (*e*) with (*f*)?

572. Copy or read and fill the blanks: To change a fraction to lower terms we divide its_______and_______by the_______number or quantity.

572*a*. If both terms of a fraction are multiplied by the same_______or quantity, the value of the fraction is_______ changed.

LESSON LVII.

573. $x + y$ is how much of Fig. 22's perimeter?

574. $x + y$ equals *what part* of Fig. 22's perimeter? $\frac{x + y}{2x + 2y}$

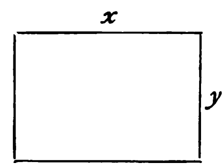


FIG. 22.

575. Reduce the answer to Ex. 574 to its lowest terms.

NOTE.—Let $x = 6$ and $y = 4$, and test the two fractions to see if they are equal.

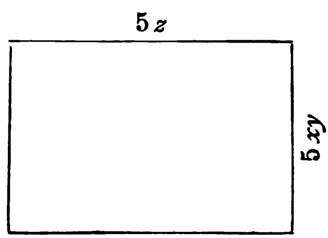


FIG. 23.

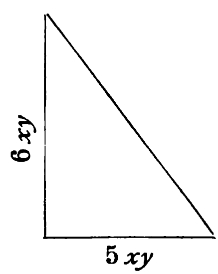


FIG. 24.

576. Find the area of Fig. 23.

577. Find the area of Fig. 24.

578. The area of the triangle is equal to what part of the rectangle?

579. Reduce the answer to Example 578 to its lowest terms.

NOTE.—Let $x = 1$, $y = 2$ and $z = 3$, and test your results.

Reduce the following fractions to their lowest terms:

580.
$$\frac{x^2}{y^2}$$

581.
$$\frac{15\,x^7y^3z^5}{85\,x^7yz^5}$$

582.
$$\frac{x^2+2\,xy+y^2}{4\,(x^2-y^2)}$$

583.
$$\frac{x^2-2\,xy+y^2}{3\,(x^2-y^2)}$$

584.
$$\frac{x^2-2\,x+1}{x^2-1}$$

585.
$$\frac{x+y}{x^2-y^2}$$

586.
$$\frac{x^4-y^4}{x^4+2\,x^2y^2+y^4}$$

587. Multiply both terms of the fraction $\frac{x+y}{x-y}$ by the quantity $x-y$.

588. Multiply both terms of the fraction $\frac{2\,x}{x+y}$ by the quantity $2\,x$.

589. Multiply both terms of the fraction $\frac{1}{x+y}$ by the quantity $(x+y)$.

590. Multiply both terms of the fraction $\frac{x-y}{2\,x-y}$ by the quantity $(x+y)$.

LESSON LVIII.

IMPROPER FRACTIONS.

(*a*).	(*b*).	(*c*).	(*d*).
$5\frac{3}{4}$	$\frac{23}{4}$	$x+\frac{x}{y}$	$\frac{xy+x}{y}$

I. What was done to the mixed number in Example (*a*) to get the improper fraction in Example (*b*)?

II. What was done to the mixed quantity in Example (*c*) to get the improper fraction in Example (*d*)?

590*a*. Copy or read and fill the blanks:

To change a mixed number or quantity to an improper fraction we multiply the whole number by the______of the fraction, and to the product add the______when the sign of the fraction is plus, and______when the sign of the fraction is______.

Change the following mixed quantities to improper fractions:

(*a*). $14-\frac{6+1}{2}$ (*b*). $\frac{28}{2}-\frac{6+1}{2}$ (*c*). $\frac{28-(6+1)}{2}$

(*d*). $\frac{28-6-1}{2}$ (*e*). $\frac{21}{2}$ (*f*). $10\frac{1}{2}$

I. See the mixed number in Example (*a*) above.

II. What is required in Example (*a*)?

III. Before we can take halves from wholes, what must be done?

IV. What was done to Ex. (*a*) to get Ex. (*b*)?

V. What was done to Ex. (*b*) to get Ex. (*c*)?

VI. What was done to Ex. (*c*) to get Ex. (*d*)?

VII. What was done to Ex. (*d*) to get Ex. (*e*)?

VIII. What was done to Ex. (*e*) to get Ex. (*f*)?

NOTE.—Repeat the same questions to the following:

(*a*). $14x-\frac{6x+1}{2}$ (*b*). $\frac{28x}{2}-\frac{6x+1}{2}$

(*c*). $\frac{28x-(6x+1)}{2}$ (*d*). $\frac{28x-6x-1}{2}$

(*e*). $\frac{22x-1}{2}$ (*f*). $11x-\frac{1}{2}$

NOTE.—It is evident that nothing new is found in these problems, they being simply examples in subtraction.

Work the following:

(*a*).

$$3x - \frac{x+2}{3} = ?$$

(*b*).

$$2y - \frac{y-4}{5} = ?$$

(*c*).

$$8y + 2 - \frac{3y+2}{4} = ?$$

Change the following to improper fractions:

591.

$$y - \frac{y}{x}$$

592.

$$x + y + \frac{x+y}{2}$$

593.

$$x + y - \frac{x+y}{2}$$

594.

$$x + y + \frac{xy}{x+y}$$

595.

$$x - y + \frac{1}{x+y}$$

596.

$$y + y + \frac{x}{y}$$

597.

$$x^2 + xy + y^2 + \frac{z^3 - x^3}{x-y}$$

598.

$$x + 3 + \frac{2x+8}{x-5}$$

LESSON LIX.

ADDITION AND SUBTRACTION.

(*a*).

$$\frac{2}{3} + \frac{3}{4}$$

(*b*).

$$\frac{8}{12} + \frac{9}{12} = \frac{17}{12}$$

(*c*).

$$\frac{a}{c} + \frac{a}{b}$$

(*d*).

$$\frac{ab}{bc} + \frac{ac}{bc} = \frac{(ab+ac)}{bc}$$

I. What was done to the fractions in Example (*a*) to get the fractions in Example (*b*)?

II. What was done to the fractions in Example (*c*) to get the fractions in Example (*d*)?

Find the sum or difference of each of the following:

599.

$$\frac{x}{y}+\frac{y}{z}=?$$

600.

$$\frac{x}{y}-\frac{y}{z}=?$$

601.

$$\frac{x}{x-y}+\frac{z}{x+y}=?$$

602.

$$\frac{x}{x+y}-\frac{z}{x-y}=?$$

603.

$$\frac{x+y}{x-4}-\frac{x+y}{x-3}=?$$

604.

$$\frac{x}{x+y}+\frac{y}{x+y}=?$$

605.

$$\frac{y}{3}+\frac{y}{2}+\frac{y}{4}=?$$

606.

$$\frac{x+y}{x-y}-\frac{x+y}{x+y}=?$$

607.

$$2x+\frac{5x}{7}+2+\frac{x}{3}=?$$

608.

$$\frac{x}{x-4}+\frac{y}{x-3}=?$$

LESSON LX.

MULTIPLYING FRACTIONS BY WHOLE NUMBERS.

(*a*).	(*b*).	(*c*).	(*d*).
$\frac{3}{4}\times 2$	$\frac{6}{4}$	$\frac{a}{b}\times c$	$\frac{ac}{b}$

I. What was done to the fraction in Example (*a*) to get the fraction in Example (*b*)?

II. Which is larger, the fraction in (*a*) or (*b*)?

III. Then what was done to the fraction in (*a*)?

IV. What was done to the fraction in Example (*c*) to get the fraction in Example (*d*)?

V. Which is larger, the fraction in (*c*) or (*d*)?

NOTE.—If in doubt let $a=3$, $c=4$ and $b=2$ and test.

VI. Then what was done to the fraction in (*c*)?

(*e*).	(*f*).	(*g*).	(*h*).
$\frac{3}{4} \times 2$	$\frac{3}{2}$	$\frac{a}{b^3} \times b$	$\frac{a}{b^2}$

I. What was done to the fraction in (*e*) to get the fraction in (*f*)? What did it do to the fraction in (*e*)?

II. What was done to the fraction in (*g*) to get the fraction in (*h*)? What did it do to the fraction in (*g*)?

609. Copy or read and fill the blanks: Multiplying the ______ or ______ the dominator of a fraction ______ the value of a fraction.

610. And from the above it must follow that: Dividing the numerator or ______ the denominator ______ the value of a fraction.

LESSON LXI.

Find the products of the following:

611. $\frac{x}{y} \times z = ?$

612. $\frac{x+3}{z} \times (x-4) = ?$

613. $\frac{x-4}{y} \times (x-8) = ?$

614. $\frac{x+y}{x-y} \times (x+y) = ?$

615. $\frac{x-8}{xyz} \times (x+3) = ?$

616. $\frac{x}{y^3} \times y = ?$

617. $\frac{x+y}{x^2-y^2} \times (x+y) = ?$

618. $\frac{x^2+y^2}{x^2-y^2} \times (x-y) = ?$

619. $\frac{x-y}{xz} \times (x-y) = ?$

620. $\frac{2x+2y}{x+y^2} \times (2x+2y) = ?$

LESSON LXII.

DIVIDING FRACTIONS BY WHOLE NUMBERS.

Find the quotients of the following:

(*a*).	(*b*).	(*c*).
$\frac{a^3}{x} \div a = ?$	$\frac{x}{a^3} \div x = ?$	$\frac{xy}{z} \div x = ?$

621. $\frac{x^2 - y^2}{z} \div (x + y) = ?$

622. $\frac{xy}{y + 4} \div (y + 3) = ?$

623. $\frac{x + y}{z + 7} \div (z - 4) = ?$

624. $\frac{(y + z)^2}{xy} \div (y + z) = ?$

625. $\frac{x^2y^2}{z - 8} \div (z - 4) = ?$

626. $\frac{(x - y)^2}{3} \div (x - y) = ?$

LESSON LXIII.

PARTITION.

(*a*).	(*b*).
$\frac{3}{4}$ of $20 = ?$	$\frac{x}{y}$ of $y^2 = ?$

I. What are we required to do in Example (*a*)?

II. What shall we do first? When we have found $\frac{1}{4}$, what shall we do? Then $\frac{3}{4}$ of 20 equals what?

III. What are we required to do in Example (*b*)?

IV. What shall we do first? When we have found $\frac{1}{y}$ what shall we do? Then $\frac{x}{y}$ of y^2 equals what?

SOLUTIONS.

$\frac{3}{4}$ of $20 = ?$	$\frac{x}{y}$ of $y^2 = ?$
$\frac{1}{4}$ of $20 = 5$	$\frac{1}{y}$ of $y^2 = y$
$\frac{3}{4} = 3$ *times* $5 = 15$	$\frac{x}{y} = x$ *times* $y = xy$

(*c*).

$$\frac{3}{4} \text{ of } \frac{3}{5} = ? \quad \frac{1}{4} \text{ of } \frac{3}{5} = \frac{\frac{3}{4}}{5}$$

$$\frac{3}{4} \text{ of } \frac{3}{5} = 3 \text{ times } \frac{\frac{3}{4}}{5} = \frac{\frac{9}{4}}{5} = \frac{9}{4} \div 5 = \frac{9}{4} \div \frac{20}{4} = \frac{9}{20}$$

(*d*).

$$\frac{a}{b} \text{ of } \frac{c}{d} = ? \quad \frac{1}{b} \text{ of } \frac{c}{d} = \frac{\frac{c}{b}}{d}$$

$$\frac{a}{b} \text{ of } \frac{c}{d} = a \text{ times } \frac{\frac{c}{b}}{d} \; 2 \; \frac{\frac{ac}{b}}{d} = \frac{ac}{b} \div d = \frac{ac}{b} \div \frac{bd}{b} = \frac{ac}{bd}$$

NOTE.—By inspection we see the same result will be obtained by multiplying the numerators together for a new numerator and the denominators together for a new denominator. This process of dividing a number into a number of equal parts to find the number in one part is PARTITION, sometimes *called* multiplying a fraction by a fraction.

627.

$$\frac{a}{b} \text{ of } \frac{c}{d} = ?$$

628.

$$\frac{x-y}{x+y} \text{ of } \frac{x-y}{x+y} = ?$$

629.

$$\frac{z}{x-y} \text{ of } \frac{b}{x-y} = ?$$

630.

$$\frac{xy}{y+3} \text{ of } \frac{yz}{y-8} = ?$$

631.

$$\frac{abc}{x+4} \text{ of } \frac{abc}{x+1} = ?$$

632.

$$\frac{c^2d^2}{x-8} \text{ of } \frac{c^2d^2b}{x-10} = ?$$

LESSON LIV.

DIVISION OF FRACTIONS

I. How many pounds of sugar can be bought for \$1 at x cents a pound? II. How many hats at \$4 each can be bought with \$12?

(*a*). $\dfrac{100}{x}$ (*b*). \$12 ÷ \$4

III. Examples (*a*) and (*b*) are examples in what?

IV. Why do we say in Example (*a*) 100 divided by x or $\dfrac{100}{x}$ when the amount given was \$1?

V. Do the dividend and divisor in Example (*b*) have the same name?

VI. If we are to divide $\frac{3}{4}$ by $\frac{2}{8}$ what shall we do? (Get them to the same name or common denominator.)

(*c*). $\dfrac{3}{4} \div \dfrac{2}{8}$ (*d*). $\dfrac{6}{8} \div \dfrac{2}{8} = 3.$

VII. When we get the fractions in (*c*) to the same name, what do we have? See Example (*d*).

VIII. What did we divide in Example (*d*) to get the answer 3? Then how many $\frac{2}{8}$ in $\frac{6}{8}$?

(*e*). $\dfrac{a}{b} \div \dfrac{c}{d}$ (*f*). $\dfrac{ad}{bd} \div \dfrac{bc}{bd}$ (*g*). $\dfrac{ad}{bc}$

IX. What are we required to do in Example (*e*)?

X. What was done to the fractions in Example (*e*) to get the fractions in (*f*)?

XI. What did we do to the fractions in (*f*) to get the answer shown in (*g*)?

(*h*). $\frac{a}{b} \div \frac{c}{d}$ (*i*). $\frac{a}{b} \times \frac{d}{c}$ (*j*). $\frac{ad}{bc}$

XII. What did we do to the fractions in (*h*) to get the fractions in (*i*)? Notice the divisor.

XIII. What did we do to the fractions in (*i*) to get the fractions in (*j*)?

XIV. How does the answer in (*j*) compare with the answer in (*g*)?

633. Copy or read and fill the blanks:

To divide a fraction by a fraction we change them to a common______and then *divide* the______. Or we may *invert* the terms of the______and *multiply* the *numerators* for a new numerator and *multiply* the *denominators* for a new *denominator*, as in partition.

LESSON LXV.

Find the quotients in the following examples (1) by changing them to a common denominator and dividing the numerators (2) by inverting the terms of the divisor and proceeding as in partition. (3) Give values to the letters and see if your two answers agree.

634. $\frac{x}{y} \div \frac{a}{b}$

635. $\frac{z}{y} \div \frac{c}{d}$

636. $\frac{a}{x} \div \frac{b}{y}$

637. $\frac{a}{d} \div \frac{x}{y}$

638. $\frac{a}{x-y} \div \frac{b}{x+y}$

639. $\frac{8-y}{8cz} \div \frac{3cx}{4z}$

640. $x - \frac{y}{2z} \div \frac{a}{b}$

641. $\frac{4b}{x+1} \div \frac{2a}{x+2}$

642. $1 + \frac{1}{x} \div \left(1 - \frac{1}{x^2}\right)$

643. $x + \frac{y-x}{1+xy} \div \left(1 - x\frac{y+x}{1+xy}\right)$

LESSON LXVI.

EQUATIONS CONTAINING THREE UNKNOWN QUANTITIES.

644. The sum of three numbers is 12. If from the sum of the first and third the second is taken, the difference is 4. If twice the third is taken from the sum of the first and second, the difference is 6. Find the numbers.

Let x = the first number,
and y = the second number,
and z = the third number.

$$\begin{aligned} (1).\quad & x + y + z = 12 \\ (2).\quad & x - y + z = 4 \\ (3).\quad & x + y - 2z = 6 \end{aligned}$$

(*a*). We may select any two of the above equations, as (1) and (2), and from them eliminate one of the unknown quantities, thus:

$$\begin{aligned} (1).\quad & x + y + z = 12 \\ (2).\quad & \underline{x - y + z = 4} \\ (4).\quad & 2x + 2z = 16 \end{aligned}$$

(*b*). What did we do to get rid of y?

(*c*). Read equation (4).

(*d*). We may now select either of the equations (1) and (2) and combine it with (3), so as to get rid of the same unknown quantity again. Taking, then, (2), we have:

$$
\begin{array}{lrl}
(2). & x - y + z = & 4 \\
(3). & x + y - 2z = & 6 \\
\hline
(5). & 2x - z = & 10
\end{array}
$$

(e). What did we do to get rid of y in this case. Read equation (5).

(f). We now have two equations containing two unknown quantities which we have learned how to solve, viz.:

$$
\begin{array}{lrl}
(4). & 2x + 2z = & 16 \\
(5). & 2x - z = & 10 \\
\hline
(6). & 4x - 2z = & 20 \\
(4). & 2x + 2z = & 16 \\
\hline
 & 6x = & 36 \\
 & x = & 6
\end{array}
$$

(g). What did we do to equation (5) to get equation (6)?

(h). We will now substitute the value of x in equation (5) as follows:

$$
\begin{array}{lrl}
(5). & 12 - z = & 10 \\
 & -z = & -2 \\
 & z = & 2
\end{array}
$$

(i). Next we will substitute the values of x and z in equation (1) as follows:

$$
\begin{array}{lrl}
(1). & 6 + y + 2 = & 12 \\
 & y = & 4
\end{array}
$$

Hence the numbers are 6, 4 and 2.

644*a*. Find the value of x, y and z.

$$
\begin{array}{lrl}
(1). & 5x - 4y + 2z = & 6 \\
(2). & 2x + 3y - 4z = & 11 \\
(3). & 3x + 2y + 5z = & 37
\end{array}
$$

$$
\begin{array}{lrl}
(1). & 5x - 4y + 2z = & 6 \\
(2). & 2x + 3y - 4z = & 11 \\
(4). & 10x - 8y + 4z = & 12 \\
\hline
(5). & 12x - 5y = & 23
\end{array}
$$

$$
\begin{array}{lrcl}
(1). & 5x - 4y + 2z & = & 6 \\
(3). & 3x + 2y + 5z & = & 37 \\
(6). & 25x - 20y + 10z & = & 30 \\
(7). & 6x + 4y + 10z & = & 74 \\
(8). & 19x - 24y & = & -44
\end{array}
$$

$$
\begin{array}{lrcl}
(5). & 12x - 5y & = & 23 \\
(8). & 19x - 24y & = & -44 \\
(9). & 288x - 120y & = & 552 \\
(10). & 95x - 120y & = & -220 \\
(11). & 193x & = & 772 \\
 & x & = & 4
\end{array}
$$

$$
\begin{array}{lrcl}
(5). & 48 - 5y & = & 23 \\
 & -5y & = & -25 \\
 & y & = & 5
\end{array}
$$

$$
\begin{array}{lrcl}
(2). & 8 + 15 - 4z & = & 11 \\
 & -4z & = & -12 \\
 & z & = & 3
\end{array}
$$

We should now be prepared to answer the following questions: See Example 644*a*.

1. Read equation (1). Read equation (2).
2. Read equation (3).
3. Which two equations were first selected?
4. What was done to (1) to get (4)?
5. What was done to get (5)?
6. Which unknown quantity was eliminated?
7. Which equation was selected to combine with (3)?
8. What was done to (1) to get (6)?
9. What was done to (3) to get (7)?
10. What was done to get (8)?
11. Which two equations were next combined?

12. What was done to (5) to get (9)?
13. What was done to (8) to get (10)?
14. What was done to get (11)?
15. In which was the value of x substituted?
16. In which were the values of x and y substituted?
17. Who can solve equations containing three unknown quantities?

LESSON LXVII.

PROBLEMS.

645. There are three numbers. If from 5 times the first, 7 times the second be taken and to the difference 6 times the third be added the algebraic sum will be 4. Or if from 2 times the first plus 4 times the second 5 times the third be taken the difference will be 6. Or if we add the first to three times the second and from the sum take 2 times the third the difference will be 10. Find the three numbers.

645*a*. What is the value of x, y and z in the following equations?

$$\begin{aligned} x + y + z &= 9 \\ 2x + 3y + 2z &= 20 \\ 2x + 4y - z &= 10 \end{aligned}$$

646. Three boys, A, B and C, have some marbles. A's added to B's equals 14. A's added to C's equals 2 more than the sum of A's and B's. B's added to C's equals 2 more than A's added to C's. How many marbles had each boy?

647. There are three numbers. If we add $\frac{1}{2}$ the first to $\frac{1}{3}$ the second, plus $\frac{1}{4}$ the third, the sum will be 62. Or $\frac{1}{3}$ the first, plus $\frac{1}{4}$ the second, plus $\frac{1}{5}$ the third equals 47. Or $\frac{1}{4}$ the first, plus $\frac{1}{5}$ the second, plus $\frac{1}{6}$ the third equals 38. Find the numbers.

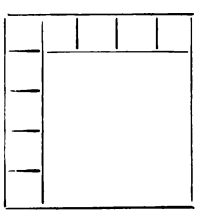
648. "Three persons A, B and C, purchase a horse for \$100, but neither is able to pay for the whole. The payments would require the whole of A's money, together with one-half of B's; or the whole of B's, with $\frac{1}{3}$ of C's; or the whole of C's with $\frac{1}{4}$ of A's. How much money has each?"

649. The sum of A's, B's and C's ages is 80 years. The sum of A's and C's ages equals 20 years more than B's age. C's age equals the sum of A's and B's age. How old is each person?

LESSON LXVIII.

SQUARE ROOT.

650. Cut out a four-inch square. Cut a strip one inch wide and nine inches long. Cut the strip into nine square inches. Place the four-inch square on your desk. Place the square inches on the edges of the four-inch square, as in the cut:



On how many edges did you place the inch squares? Then, what kind of a figure did you have? On how many edges of a square must you build squares to keep it a square?

If we had a six-inch square, how many square inches would we use to keep it a square? (13–Ans.) If an eight-inch square? If a ten-inch square?

When we have a square, how do the number of rows of square units on its surface compare with the number of square units in each row? Are they *always* the same?

When we say "square 3," what do we mean? (We mean a square surface 3 by 3, or a surface containing 9 square units.)

What is the square of 6? —of 8? —of 9?

What is the $\sqrt{36}$? —of $\sqrt{64}$? —of $\sqrt{81}$?

What is the length and width of the smallest square whose square root can be expressed with one figure? (1 by 1.) What is the largest square whose root can be expressed by one figure? (9 by 9.) 9^2 = what? Then, if a square contains more than 81 square units, what do you know about its root? (Its root must be expressed by more than one figure.)

What is the square of 10, or of $10^2 = ?$ (100.)

$$11^2 = 121$$
$$12^2 = 144$$
$$20^2 = 400$$
$$60^2 = 3600$$
$$99^2 = 9801$$

The square root of any number expressed with three or four figures will be expressed by how many figures? (Two.) A number expressed by two figures is made up of what? (Tens and units.)

If we have a square containing 144 square inches, what must its root contain? (Tens and units.) How many tens can the root contain? (Not more than one; because the square of 2 tens, or 20, is 400, and this square has but 144 square inches.)

If from this square containing 144 square inches we cut a square 10 by 10 inches, how many square inches will we take? (Ten rows of ten square inches, or 100 square inches.) How many square inches will remain? (144 less 100, or 44.) If we place the square inches that remain on two edges of the ten-inch square, how many will we use in going around once? (Twenty-one.) Have we used all we had? (No, we had 44, and have used but 21.) If we

use 21 in going around once, how many times can we go around with 44? (As many times as there are 21 square inches in 44 square inches, or 2.) Let us see if this is true.

```
     144(12
     100
     ---
 21)  44
      --
      20
      24
      --
      44
```

On the upper edge we will use two rows of 10 square inches, or 20 square inches. On the side we will use two rows of 12 square inches, or 24. In all we have used 20 plus 24, or 44, just what remained; so we know the root must be one ten and two units, or 12.

What is the square root of 529?

```
     529(23
     400
     ---
 41) 129
     ---
      60
      69
     ---
     129
```

We know the root will be made up of tens and units. The largest ten the root *can* have is 2. The square of 2 tens is 400; so, if we cut out a square 20 by 20, we cut out 400 square inches. There are left 129 square inches. If we use these to go around two edges of the square 20 by 20, we will use 41 of them in going once around. We can go around as many times as there are 41 square inches in 129 square inches, or 3. *Proof:*—On the upper edge we will use three rows of 20 square inches, or 60, and on the side three rows of 23 square inches, or 69. In all, 60 square inches plus 69 square inches, or 129 square inches, just what we had left; so we know 3 is the unit figure.

```
     144(12
     100
     ---
 20)  44
      40
      --
       4
```

Notice that if we use *twice* the tens of the root for a trial divisor, the remainder after dividing is the square of the units.

This is always true if the number is a perfect square.

LESSON LXIX.

What is the square root of each of the following?

Example	651.	484.	Example	656.	144.
"	652.	576.	"	657.	169.
"	653.	961.	"	658.	2916.
"	654.	1024.	"	659.	9801.
"	655.	1600.	"	660.	5329.

661. What is the square root of 15129?

```
       15129(123
       14400
       -----
241)    729
        ---
        360
        369
        ---
        729
```

NOTE.—There are 12 tens in the root. 12 tens $(120)^2 = 14400$, and 15129 less 14400 = 729.

Once around and we use 241. 729 ÷ 241 = 3. *Proof:*—Upper edge will take three rows of 120 each, = 360. The side will take three rows of 123 each, = 369; and 360 plus 369 equals 729, just what we had to use. Hence the square root is 123.

LESSON LXX.

(*a*).	(*b*).	(*c*)
$.1^2 = .01$	$.01^2 = .0001$	$.001^2 = .000001$
$.9^2 = .81$	$.09^2 = .0081$	$.009^2 = .000081$

I. What was the first number squared in (*a*)? What is the square root of .01? —of .81?

II. What is the square root of .0001? See (*b*).

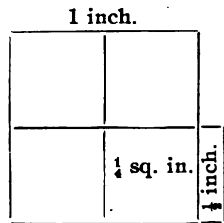
III. What is the square root of .0081? —of .000001? —of .000081? See (*c*).

IV. How do the number of places of any decimal compare with its square root?

Extract the square roots of

662. .09	666. .0256
663. .16	667. .2025
664. .0225	668. .1024
665. .0625	669. 7 to three places.

Note.—It is often a mystery to children how the square of a fraction can be less than the fraction, *e.g.*, $(\frac{1}{2})^2 = \frac{1}{4}$, $(\frac{3}{4})^2 = \frac{9}{16}$, etc. Draw a square inch, divide it into four equal parts and notice that the squares, measuring $\frac{1}{2}$ inch on each side, contain but $\frac{1}{4}$ of a square inch. See illustration.



Square the following:

(1).	(2).	(3).	(4).	(5).
$\frac{3}{4}$	$\frac{5}{6}$	$\frac{7}{8}$	$\frac{2x}{4}$	$\frac{2}{x^2y^2}$

Write the answers to the following:

(6). $\left(\frac{1+y}{1+x}\right)^2 = ?$ (8). $\left(\frac{2x^2y}{3ab^2}\right)^2 = ?$ (9). $\left(\frac{x+3}{y-1}\right)^2 = ?$

(9). $\frac{\sqrt{5}}{25} = ?$ (10). $\frac{\sqrt{36}}{42} = ?$ (11). $\frac{\sqrt{4x^4y^2}}{9a^2b^4} = ?$ (12). $\frac{\sqrt{(x+y)^2}}{4x^2} = ?$

LESSON LXXI.

SQUARE ROOT OF POLYNOMIALS.

(*A*).

$$\begin{array}{l|l} x^2 + 2xy + y^2 & \underline{x+y} \\ \underline{x^2} & \\ \underline{2x} \;\; & 2xy + y^2 \end{array}$$

Notice the polynomial is arranged according to the power of x, the highest power first, as in division, multiplication, etc.

(1). What is the first term? What is its square root? Make this square root x, the first term of the root of the polynomial. Square it and take

the square from the first term of the polynomial. We have taken out a square x by x, and have $2xy+y^2$ units to build around it. Once around will take at least $2x$ (see square root of numbers). $2x$, then, is our trial divisor, for if we take $2x$ in going once around we can go around as many times as there are $2x$'s in $2xy+y^2$, or $+y$ times. Adding this term of the root (y) to the trial divisor, as shown here in (B), we get a complete divisor, $2x+y$. Multiply the complete divisor by the last term of the root (y) and subtract the product from the remainder, and bring down the next terms, if any, and proceed as before. In this case there was nothing to bring down.

(B).

$$\begin{array}{l|l} \underline{2x+y} & 2x+y^2 \\ & 2x+y^2 \end{array}$$

Supply y with your pencils in (A) and multiply as in (B). You see it is the same thing.

Next we will take a polynomial of six terms, and you may tell what is done to get its square root, by answering the questions below:

$$\begin{array}{lll}
 & x^2+2xy+y^2+2xz+2yz+z^2 & \underline{\mid x+y+z} \\
 & x^2 & \\
\underline{2x+y} \mid & \overline{\quad 2xy+y^2} & \\
 & \quad 2xy+y^2 & \\
\underline{2x+2y+z} \mid & \overline{\qquad 2xz+2yz+z^2} & \\
 & \qquad \underline{2xz+2yz+z^2} &
\end{array}$$

Hence $\sqrt{x^2+2xy+y^2+2xz+2yz+z^2}=x+y+z$

(a). After the polynomial was arranged, what did we do first?

(b). Where did we get the x^2 which is subtracted from the polynomial?

(c). Where did we get the $2x$ term of the trial divisor? Where the y term?

(*d*). Where did we get the $2xy + y^2$ that is *subtracted from?* Where the $2xy + y^2$ that is subtracted?

(*e*). Where did we get the $2x + 2y$ terms of the second trial divisor? Where the z term?

(*f*). Where did we get the $2xz + 2yz + z^2$ that is *subtracted from?* Where the $2xz + 2yz + z^2$ that is subtracted? What is the square root?

670. Extract the square root of $e^2 + 2ef + f^2 + 2eg + 2fg + g^2$.

671. Extract the square root of $a^2 + 2ab + b^2 + 2ac + 2bc + c^2$. What is the square root of $9 - 30x^2 + 25x^4$?

672. Find the square root of $9x^2 + 12x + 12xy + 4y^2 - 8y + 4$.

673. The area of a square lot equaled $49x^2y^2 - 24xy^3 + 25x^4 - 30x^3y + 16y^4$, to what was the length of each side equal?

LESSON LXII.

QUESTIONS ON CHART II.

NOTE.—There is nothing new or difficult in these questions. You have simply to use what you have learned. Call $3\frac{1}{7}$ times the diameter of a circle its circumference.

674. Find the circumference of Fig. 1.
675. What is the area of Fig. 1?
676. What is the circumference of Fig. 2?
677. What is the area of Fig. 2?
678. What is the perimeter of Fig. 3?
679. What is the area of Fig. 3?
680. What is the perimeter of Fig. 4?
681. What is the area of Fig. 4?
682. What is the area of Fig. 5?
683. What is the area of the 4 walls and ceiling of a room $x + y$ long and $x + y$ wide, and $x + y$ high?

CHART II.

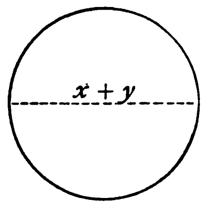


FIG. 1.

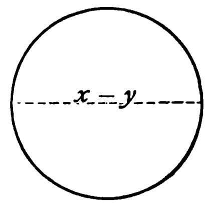


FIG. 2.

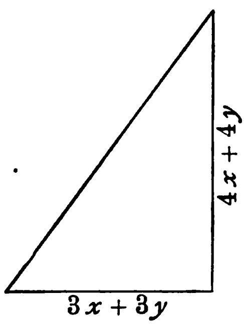


FIG. 3.

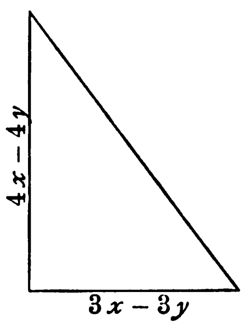


FIG. 4.

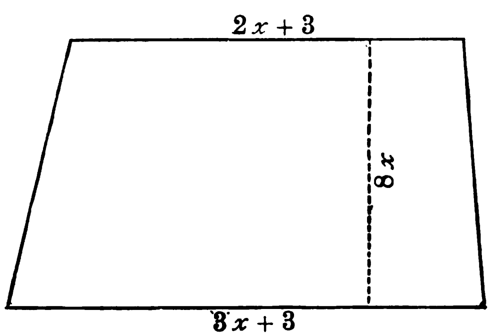


FIG. 5.

LESSON LXXIII.

QUESTIONS ON CHART III.

684. How many square feet on the side walls of a room of which Fig. 6 is a plan?

685. How many square feet on the ceiling of the same room the altitude of the triangle being x feet?

686. How many square feet on the side walls and ceiling of this room?

687. How many square feet on the sides of a room of which Fig. 7 is a plan?

688. How many square feet on the sides and ceiling of the room? (Fig. 7.)

689. Fig. 8 is the plan of a room. How many square feet on its four walls?

690. What is the area of the ceiling if the area of the 4 walls and ceiling equals $45x^2 + 165x + 104$. Ex. 689.

LESSON LXXIV.

QUESTIONS ON CHARTS IV. AND V.

691. What is the area of the ceiling of a room of which Fig. 9 is a plan?

692. What is the area of the four walls of the room of which Fig. 9 is a plan?

693. What is the volume of Fig. 10? Give values to your letters ($x = 1, y = 1$) and see if your long answer is correct. It is evident there are 8 cubic inches.

694. Mr. A owned a lot that was oblong in shape, its area was $x^2 + 10x + 16$ sq. rods. How much longer than wide was the lot?

695. What was the length of each side of a square lot whose area was $64x^2 + 48xy + 9y^2$ square rods?

696. What is the perimeter of the triangle shown in Fig. 11?

697. Figs. 12, 13 and 14, Chart VI, show the squares of the sides of a triangle. What is its perimeter? If $x = 1$ and $y = 2$, construct the triangle on the blackboard.

CHART III.

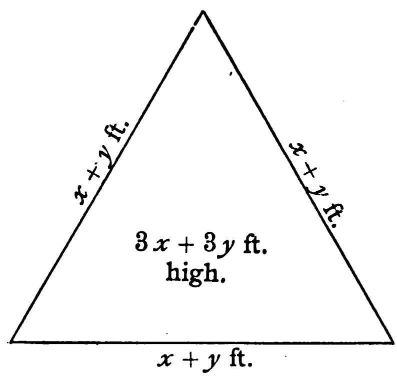


FIG. 6.

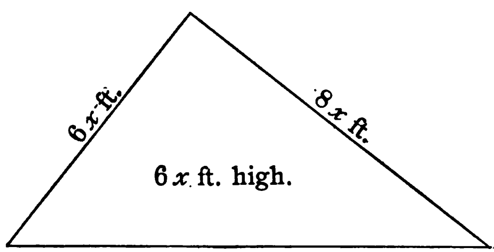


FIG. 7.

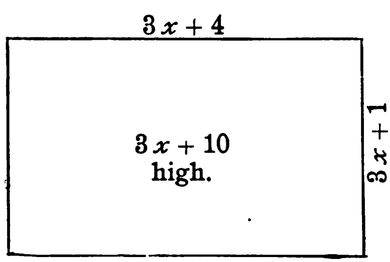


FIG. 8.

CHART IV.

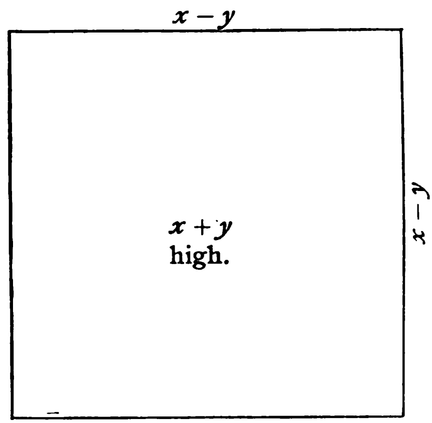


FIG. 9.

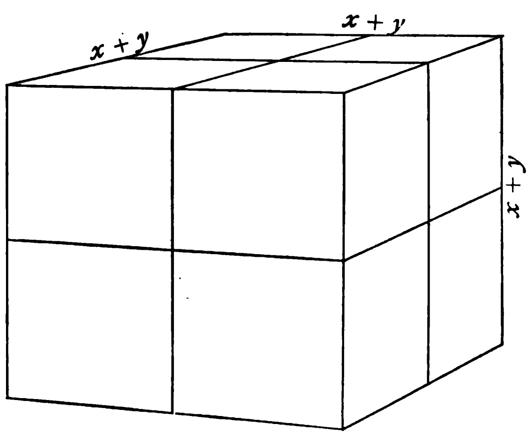


FIG. 10.

CHART V.

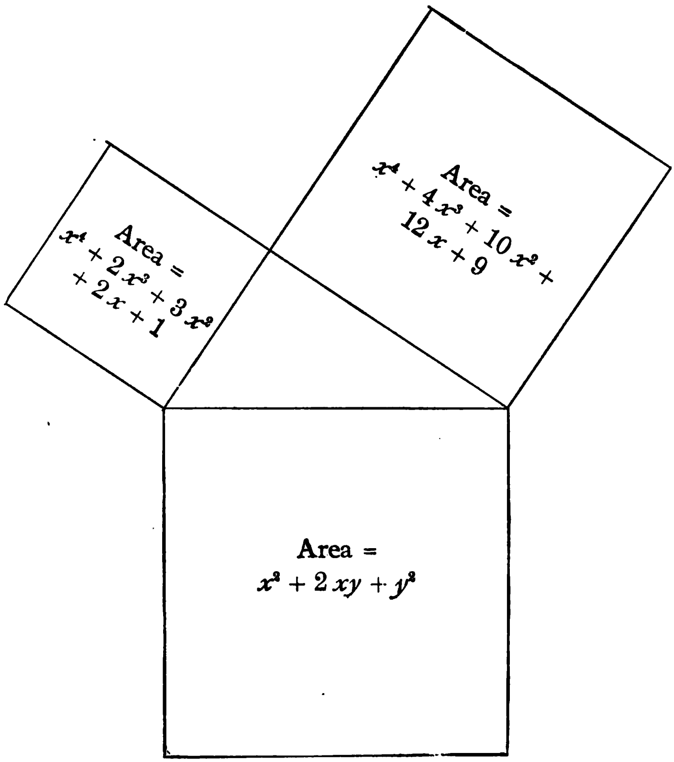


FIG. 11.

CHART VI.

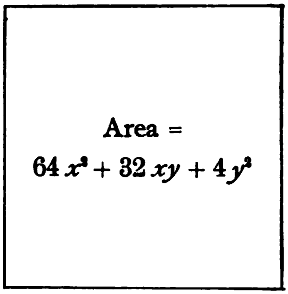


FIG. 12.

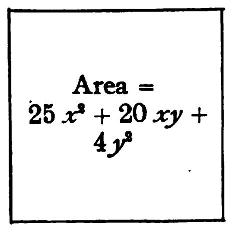


FIG. 13.

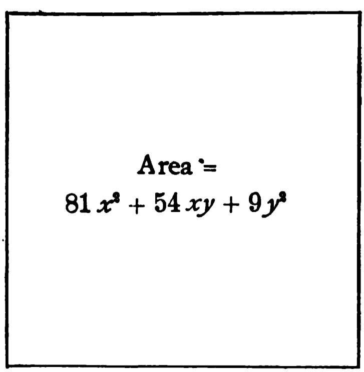


FIG. 14.

CHART VII.

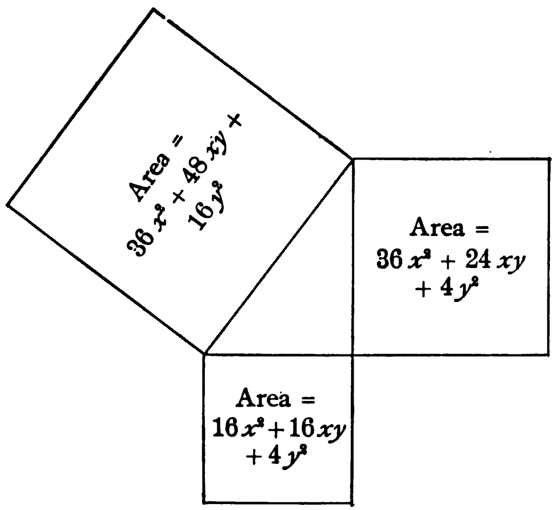


FIG. 15.

698. To what is the perimeter of the triangle in Fig. 15 equal?

LESSON LXXV.

MISCELLANEOUS PROBLEMS.

699. Add $3x^2+2y+3z+7y$, $+6z+3x^2-9z-y+6x^2+3y+17z$, $-8x^2+4z^2-2y^2+z$, $+7x^2+4y+3z$.

700. What must be subtracted from $5a^3-4a^2b+3b^2c$ to leave $+7a^3-7a^2b+11b^2c$?

701. From $4xy-cd+3x^2$ take $5xy-4cd+3x^2+5b^2$.

702. Take $12x^2y^2+12xy+8xyz+4abc-z$ from $18x^2y^2$.

703. What number multiplied by $a+b$ will obtain $a^3b^2+ac^2d+a^2b^3+bc^2d$ for a product?

704. The product of two quantities is $a^4+4a^3x+6a^2x^2+4ax^3+x^4$. One of the quantities is $a+x$; what is the other?

705. What number divided by $2x+2$ will obtain $3x-4$ for a quotient?

706. We must divide $10x^4+48x^3y+13x^2y^2+4xy^3-15y^4$ by what quantity to obtain $2x^2+8xy-5y^2$ for a quotient?

707. The sum of the two parallel sides of a trapezoidal lot is 48 rods. The longer side of the two is twice as long as the shorter. The altitude is 3 times as long as the shorter parallel side. How many acres does the lot contain?

708. Increase the length of a given rectangle 2 ft. and its width 1 ft., and its area is increased 12 sq. ft. Diminish the length 3 ft., and the width 2 ft., and its area is diminished 11 sq. ft. What is the perimeter of the rectangle?

709. In a given equation $\frac{x-1}{3}-\frac{x+4}{5}$ is equal to the following: $15-\frac{x+3}{4}$. What is the value of x?

710. John had a certain number of marbles. 3 times John's marbles less 5, divided by 2 equals Will's. The sum of both is equal to $12 \frac{2x-4}{3}$. How many has each boy?

711. The volume of a cubical box is equal to $8x^9 + 12x^6 + 6x^3 + 1$. To what are its dimensions equal?

712. To what are the dimensions of a cubical box equal whose volume is equal to $8x^6 - 36x^4y^2 + 54x^2y^4 - 27y^6$?

713. The area of a square is equal to $1 + 2xy + x^2y^2$. To what are its length and breadth equal?

714. To what are the length and breadth of a square equal whose area is equal to $x^2y^2 + 2xy^2z + y^2z^2$?

715. To what are the length and breadth of a rectangle equal whose area is equal to $a^2 - 7a + 12$?

716. To what would the length and breadth of a rectangle be equal whose area is equal to $x^2 + 7x + 12$?

717. What if the area of the last rectangle had been equal to $a^2 + a - 12$?

718. The $\sqrt{x^2 - 2xy + y^2}$ = what?

719. If the dimensions of a cube are $3x + 3y$, what is its volume?

720. What is the volume of a cube whose dimensions are $3x - 3y$?

721. What two quantities multiplied together will give $4x^4y^2 - 12x^3y^3 + 9x^2y^4$?

722. The product of two quantities is equal to $x^2y^2 + 10xy + 25$, What are the two quantities?

723. The sum of the distances which A, B and C have traveled, is 62 miles, A's distance is equal to 4 times C's added to twice B's; and twice A's added to 3 times B's is equal to C's multiplied by 17. How far has each man traveled?

724. Henry can do a piece of work in 5 days. George can do the same work in 3 days. How long will it take them both working together to do it?

725. The diameter of a circle is $7x + 7y$ feet. What is its area?

726. If x – the fraction one-eighth is equal to $\frac{5}{6}x$ plus the fraction $\frac{1}{12}$; what is the value of x?

727. *a* plus *b* equals 9, *a* plus *c* equals ten and *b* plus *c* equals one less than a dozen; what is the value of each?

728. B is $\frac{5}{6}$ as old as A. 5 years ago he was $\frac{4}{5}$ as old. What is the age of each person?

729. The square of the sum of two numbers equals 100. The square of the difference of the same numbers equals 4. What are the numbers?

730. The sum of two numbers equals 45. If the greater of the two be divided by the less the quotient will be 2. What are the numbers?

731. The sum of two fractions equals $1\frac{1}{3}$. Their difference equals $\frac{1}{3}$. What are the fractions?

732. Two men, A and B, buy a horse together for $140. A said, "If B will give me $\frac{2}{3}$ of his money, I can then buy it alone." B said, "If A will give me $\frac{1}{3}$ of his money, I can buy it alone." How much more money had A than B?

733. Mary has a certain number of marbles. She buys two more. If now she only had twice as many, the whole number would equal 14. How many has she?

734. Change the fraction $\frac{4x^3y^2}{6x^4}$ to its simplest form.

735. To what is the fraction $\frac{2(x+y)}{3x(x+y)}$ equal in its simplest form? Is its value changed?

736. Thomas has a certain number of marbles. Henry has 16. If we divide the number that Thomas has by 4,

we shall get the same quotient that is obtained by dividing Henry's by the number that Thomas has. How many has he?

737. The sum of three numbers is 9. As the first added to 2 times the second, plus 4 times the third equals 15, and 3 times the second, plus 9 times the third added to the first equals 23, what are the numbers?

738. A farmer plants 5476 trees, and has as many rows of trees as there are trees in each row. How many trees are there in each row?

739. To the $\sqrt{16x^2y^2}$ add the $\sqrt{64x^2y^2}$ and from the sum take the square root of $144x^2y^2$; what remains?

740. There are three consecutive numbers whose sum is 117. What are the numbers?

741. A buys some 3¢ stamps and 12 more than twice as many 2¢ stamps, paying for all $1.36. How many of each did he buy?

742. A number that is written with two digits has in unit's place a digit 2 times the digit in ten's place. The entire number less 8 equals five times the digit in unit's place. What is the number?

743. If the sum of two numbers be taken from 188 the difference equals 110. Or if the difference of the two numbers be taken from 124 the difference equals 110. What are the numbers?

744. Take from the sum of two given numbers their difference and the remainder equals 36. . Find the numbers.

745. A man gave his sons James and John each a certain sum of money. One-half of John's money equals $\frac{3}{4}$ of James's money and the sum of their money is $30. How many dollars has each?

746. A man has 3 houses. The value of the first and second is $5600. The second is worth $700 more than

the third. All three together are valued at $7400. What is the value of each house?

747. A and B together have 640 sheep. One-sixth of A's sheep is equal to one-half of B's. How many sheep did each have?

748. A man walks from A to B, which is 25 miles farther than $\frac{1}{2}$ the distance from A to C, and 15 miles less than the distance from A to D. The distance from A to B plus the distance from A to C minus the distance from A to D equals 5 miles. How far is each place from A?

749. A had 84 cents and B had 48 cents. B gave A a certain sum when A had 5 times as much as B. How much did B give A?

750. A post is $\frac{1}{6}$ in the mud, $\frac{1}{3}$ in the water, and 9 feet above the water. How long is it?

751. The perimeter of a rectangle equals 48 feet. The rectangle is twice as long as it is wide. How long is each side?

752. A woman had a basket of eggs. $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of them, added to one-fourth of them, was equal to 1 and one-half dozen. How much was her eggs worth at 10¢ per half-dozen?

753. John has a certain number of marbles; he buys 2 more and then 3 times the number is equal to 15. How many had he at first?

754. I have fifty cents in dimes, nickels, and pennies. The number of coins is 16, and there are half as many dimes as nickels. How many coins of each kind have I?

755. A class of girls and boys is divided into three sections. In the first there are $\frac{1}{4}$ of the girls and $\frac{1}{5}$ of the boys, which make 17 less than $\frac{1}{2}$ the class. In the second division there are $\frac{1}{3}$ of the girls and $\frac{1}{4}$ of the boys, making, together, 24. The remainder are in the third division. How many boys and girls in the class?

756. There is a fraction which if its numerator be doubled and seven be added to its denominator equals $\frac{1}{2}$. Or if 7 be added to its numerator and its denominator be doubled it equals 1. Find the fraction.

EXERCISE I.

TESTS CONTAINING ONE, TWO OR THREE UNKNOWN QUANTITIES.

1. Three times a certain number equals itself added to twenty. What is the number?

2. Three times a certain number equals 2 times itself added to 5. What is the number?

3. Four times a given number + 5 equals 5 times itself less 1. Find the number.

4. Three times a certain number plus 10 equals itself added to 20. Find the number.

5. If from a given number 2 be taken and the difference be multiplied by 3 the product will equal two times the difference between itself and 1. What is the number?

6. 5 times a known number added to 3 times itself equals 10 less than half of one hundred. Find the number.

7. Three times a certain number added to 40 equals ten, ten, double ten, forty-five and fifteen. What is the number?

8. There is a number from which if we take the difference between $\frac{1}{2}$ itself and 20, we shall still have 45 left. What is the number?

9. If 3 times a given number of days plus 2 times the same number of days be added to one, the sum will equal the number of days in March. What is the number of days given?

10. The number 19 is eleven less than 3 times a certain number added to 3. What is the number?

EXERCISE II.

1. One-half a given number, plus its $\frac{1}{3}$, equals its $\frac{1}{6}$ added to 12. Find the number.

2. There is a certain number, the sum of whose $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{3}$ equals one and one-half dozen. What is the number?

3. If the $\frac{1}{4}$ of a certain number be taken from its $\frac{1}{3}$, the difference equals 3 less than its $\frac{1}{6}$. Find the number.

4. Henry had some marbles. Jessie had $\frac{1}{3}$ as many and together they had 28. How many had each?

5. Emma Lou had some marbles. She gave Sid $\frac{1}{5}$ and Frank $\frac{1}{7}$ of them, and still had 23. How many had she at first?

6. Johnson had a number of hens. He bought four more; $\frac{1}{3}$ of the sum was equal to $\frac{3}{8}$ of what he had at first. How many had he?

7. Frank worked 1 year, but for three different men, A, B and C. He worked twice as many months for B as for A and $\frac{1}{3}$ as many months for C as for the other two. How long did he work for each man?

8. The sum of 2 numbers is 54, one of them being twice as large as the other. What are the numbers?

9. The sum of two numbers is 42, one of them being $\frac{1}{2}$ as great as the other. Find the numbers.

10. There is a number from which if we take 13, one-fourth the difference is 5. What is the number?

EXERCISE III.

1. If from 5 times a certain number 21 be taken the difference will be equal to 7 times the number less 31. Find the number.

2. If from 5 times a given number the quantity $(2x+3)$ is taken, the difference is equal to 2 times the number plus 7. What is the number?

3. Take the quantity $(2x - 3)$ from 4 times a certain number and the difference equals four tens added to 3. What is the number?

4. The quantity $3(4x - 27)$ is equal to 6 times 23 minus a given number, diminished by 3 times the given number. Find the number.

5. What number diminished by its $\frac{1}{3}$ plus its $\frac{1}{4}$ is equal to 10?

6. What number diminished by its $\frac{1}{6}$ less its $\frac{1}{12}$ is equal to 35?

7 George had a number of apples. He gave Henry $\frac{1}{6}$ and William $\frac{1}{4}$ of them and still had 28. How many had he at first?

8. Kitty had a number of 2¢ postage stamps; she used $\frac{1}{5}$ of them on letters and $\frac{1}{10}$ of them on newspapers and still had 42 left. How many letters and how many papers did she mail?

9. A man bought a coat and a vest; the coat cost him 3 times as much as the vest, less \$3. They both cost \$17. What was the cost of each?

10. If from a given number 2 be taken $\frac{1}{4}$ the difference equal $\frac{5}{8}$ of the number. Find the number.

EXERCISE IV.

1. The sum of two numbers is 84. Their difference is 8. What are the numbers?

2. There are two numbers. If 2 times the first is added to three times the second, the sum is 29. Or if 3 times the first is added to 2 times the second the sum is 36. Find the two numbers.

3. Two girls, Helen and Lilian, had 53 oranges. Helen gave Carrie $\frac{1}{7}$ of hers and Lilian gave Carrie $\frac{1}{8}$ of hers, when Carrie had 7. How many had Helen and Lilian?

4. Two numbers are to each other as 5 is to 10. Their sum is 18. What are the numbers?

5. The sum of two numbers is 8. Four times the first less the second equals 7. Find the two numbers.

6. There are two numbers. Take $\frac{3}{5}$ of the second from the first and you will have 6. Or from $\frac{2}{3}$ of the first take $\frac{1}{5}$ of the second and you will have 9. What are the two numbers?

7. The square of the greater of three numbers equals 16; its root is the difference between the other two numbers. The sum of the same two numbers is 20. What are the numbers?

8. One-half of Madge's apples taken from $\frac{1}{3}$ of Annie's apples equals 3, and $\frac{1}{2}$ of Madge's apples added to $\frac{1}{3}$ of Annie's apples equals 9. How many apples has each girl?

9. One added to the numerator of a given fraction makes it equal to 1, and one added to its denominator make it equal to $\frac{2}{3}$. What is the fraction?

10. Two numbers whose difference equals 16 are to each other as 15 is to 3. What are the numbers?

EXERCISE V.

1. The sum of two numbers equals a. Their difference equals b. What are the two numbers? Add the numbers and see if the sum is a.

2. Grace has one pencil more than May. If we add to May's 3 times Grace's we shall have 15. How many had each?

3. A and B are traveling in the same direction, A having 10 miles the start of B. A is walking at the rate of 3 miles an hour and B at the rate of 5 miles an hour. If they started at 1 o'clock, what time will it be when B overtakes A?

4. Amy has a certain number of cents. Belle has 3 times as many. They have together c cents. How many cents has each?

5. One-half of A's money added to $\frac{1}{3}$ of B's equals \$$a$, and 3 times A's less 2 times B's equals \$$b$. How much money has each?

6. The sum of 3 numbers is 6. The first less the second equals 1. The third added to the first equals 5. Find the numbers.

7. The sum of three numbers is 12. The first less the second equals 1. The third added to the first equals 9. Find the numbers.

8. The sum of three numbers equals 19. The first less $\frac{1}{2}$ the second equals 2. The first added to $\frac{1}{4}$ the third equals 7. What are the numbers?

9. The sum of two numbers is 27; $\frac{1}{3}$ of the first equals $\frac{2}{3}$ of the second. Find the numbers.

10. The sum of two numbers is b, the second is $\frac{1}{2}$ as large as the first. Find the value of each.

EXERCISE VI.

1. In a certain village there were twice as many Americans as Englishmen and $\frac{1}{3}$ as many Swedes as Germans. If there had been 60 more Germans their number would have been $\frac{2}{3}$ of the number of Englishmen. If the number of Americans had been diminished by 100, their number would have been 10 times the number of Swedes. How many people were in the village?

2. The number of hundred square miles in New Hampshire is a number of two digits. The sum of the digits is 12. If 54 is subtracted from the number the digits will be reversed. How many square miles in New Hampshire?

3. I have a cylinder; $\frac{1}{2}$ its circumference is 4 inches longer than $\frac{1}{3}$ its altitude. The perimeter of the rectangle which the cylinder would make if opened out is 86 inches. Find the area of the cylindric convex surface.

4. The sum of three numbers equals a. The first less the third equals b. The first plus the second equals c. Find the value of each.

NOTE.—Test your answer: $a = 7$, $b = 3$ and $c = 6$, $x = 4$, $y = 2$ and $z = 1$.

5. There are three numbers whose sum is 30. The third less the first equals 10 and the sum of the first and second equals 15. Find the numbers.

6. The product of two quantities is $x^3 - xy^2 + x^2y - y^3$. One of the quantities is $x + y$. What is the other?

7. The answer to the sixth example is the product of what two quantities?

8. The quantity $z^2 - 2z - 48$ is the product of what two quantities?

9. The area of a triangle equaled $4x^2 - 4y^2$. The area of a rectangle equaled $4x^2 - 8xy + 4y^2$. The area of the rectangle is equal to what part of the area of the triangle?

10. What number is that which if its $\frac{1}{4}$ is taken from its $\frac{1}{3}$ the difference equals 4?

EXERCISE VII.

1. Said A to B, "How much money have you?" Said B, "If I had $\frac{1}{3}$ as much more and \$4 I should have \$100." How much had B?

2. In a room $\frac{1}{3}$ of the people are German, $\frac{1}{6}$ are French, $\frac{1}{4}$ are English, 16 are American, and 8 are Spanish. How many of each are there in the room?

3. A post stands $\frac{1}{3}$ of its length in mud, $\frac{1}{4}$ in water, and 15 feet out of the water; how long is it?

4. Launcey has \$50; $\frac{3}{5}$ of his money + \$40 is $\frac{7}{8}$ of what William has. How much has William?

5. If a post stands $\frac{1}{5}$ in the mud, $\frac{1}{4}$ in water, and 22 feet out of the water, what is the length of the post?

6. Another post is $\frac{1}{10}$ of its length in the mud, $\frac{1}{5}$ in the water, and 21 feet out of the water. How long is this post?

7. There is a certain number to which if we add its $\frac{1}{2}$, its $\frac{1}{4}$ and 20 more, it will be doubled. Find the number.

8. A man being asked how many bushels of oats he had, answered, that if he had as many more bushels, $\frac{1}{2}$ as many more and $2\frac{1}{2}$ bushels, he should have 100. How many bushels had he?

9. A man being asked his age, said, that if its $\frac{1}{9}$ and its $\frac{1}{5}$ were added to it, the sum would be 59. How old was he?

10. A man when asked how many sheep he had, said that in one pasture there was $\frac{1}{2}$ of them, in another $\frac{1}{4}$ of them, in another $\frac{1}{6}$ of them, and in another only 3. How many had he?

EXERCISE VIII.

FRACTIONS.

1. There are two fractions; the numerator of the first is $x + 5$ and of the second $x + 2$. The denominator of the first is $x + 4$, and of the second $x + 3$. What is the sum of the two fractions?

2. If the second fraction (Example 1) is taken from the first, what is the remainder?

3. There are two other fractions; the numerator of the first is $x - 8$; its denominator is $x + 2$. The numerator of the second is $x - 4$; its denominator is $x + 4$. What is the sum of the fractions?

4. Find the difference between the two fractions in Example 3.

5. From a fraction whose numerator is $x-y$ and whose denominator is $x+y$, take a fraction whose numerator is $x+y$, and whose denominator is $x-y$.

6. Find the sum of the two fractions in example 5.

7. The sum of two fractions is $\frac{2y^2+13y+22}{y^2+6y+8}$. One of the fractions is $\frac{y+3}{y+4}$. What is the other fraction?

8. The quotient of two fractions is $\frac{ad}{bc}$, the divisor is $\frac{c}{d}$. What is the dividend?

9. Minnie had a apples. She gave Jennie $\frac{1}{c}$ of them, and then gave May $\frac{b}{d}$ of what was left. How many did May receive?

10. The quotient of two fractions is $\frac{x^2-y^2}{a^2}$ the divisor is $\frac{a}{x-y}$. What is the dividend?

EXERCISE IX.

1. John had a marbles and Henry had c marbles. John gave $\frac{1}{b}$ of his to William, and Henry gave $\frac{1}{d}$ of his to him. How many did William then have?

2. Harry had x apples. He gave $\frac{1}{y}$ of z to John. How many had he left?

3. Sara had x oranges. She gave Angela $\frac{1}{4}$ of them and Edna $\frac{1}{3}$ of them. How many did she give away.

4. How many oranges did Sara have left? Ex. 3.

5. Sara gave Norma $\frac{1}{8}$ of what she had left. How many did she give Norma?

6. Ola had 3 more oranges than Norma. How many did Ola have? See Ex. 5.

7. George had y sticks of candy. He gave Don $\frac{1}{2}$ of them, Roy $\frac{1}{6}$ and Curtis $\frac{1}{3}$. How many had he left?

8. Roy gave $\frac{1}{2}$ of his candy (see example 7) to Percy. How many did Percy have? Cleon had as many as Percy. How many did they have together?

9. Louise had $\frac{1}{4}$ as much candy as Don. How much did she have? Ex. 7.

10. Carrie had x jacks and gave them to Lois who had y before. Lois then gave $\frac{1}{2}$ of hers to Marion. How many did she give Marion?

EXERCISE X.

1. A class was told to draw a number of rectangles. Isabella drew one that contained $x^2 + 2xy + y^2$ square inches. How long was it?

2. Nellie drew one which contained $x^2 - 2xy + y^2$ square inches. How wide was it?

3. Evelyn drew one that contained $y^2 - 7y + 12$ square inches. How long and wide was it?

4. Murray drew one that contained $4x^2 + 14x + 12$ square inches. How long and wide was it?

5. Robert drew one that contained $4x^2 - 4y^2$ square inches. How long and wide was it?

6. The area of the one Jack drew equaled xy square inches. How long and wide was it?

7. The area of the one drawn by Leslie equaled $36y^2 - 36z^2$ square inches. What were its dimensions?

8. The area of Edith's equaled $9x^2 + 12xy + 4y^2$. What was its length?

9. The area of Millie's equaled $x^3 + 3x^2y + 3xy^2 + y^3$ square inches. What was its width if its length was $x^2 + 2xy + y^2$?

EXERCISE XI.

1. What is the difference between $x - y + z$ and $-y - z$?

2. Add $x - y + z$ to $-y - z$.

3. What must be added to $-2x^2 + 16xy - 7y^2$ to obtain $3x^2 + 7xy - 2y^2$?

4. Multiply both members of the following equation by -1. $-x + 4y = -21 - 6$.

5. The area of a rectangle drawn by Andrew equaled $x^3 - 3x^2y + 3xy^2 - y^3$. To what was its length equal if its width was $x - y$?

6. A merchant bought an equal number of hats and caps for $217. He paid $5 each for the hats and $2 each for the caps. How many of each did he buy?

7. What number is that, $\frac{1}{4}$ of which is greater than $\frac{1}{5}$ of it by 3?

8. What number is that, to which 10 being added, $\frac{3}{4}$ of the sum will be 36?

9. The minuend in a certain problem is $y^3 - 3y^2 + 2z - 5$; the difference is 1. What is the subtrahend?

10. A's kite string added to B's makes one 317 feet long; but B's is 36 feet longer than A's. What is the length of each string?

11. How many gills in a quarts? How many ounces in b pounds? How many cents in c dollars? How many mills in d cents? How many dollars in e cents?

12. The sum of two numbers equals the number of quarts in a bushel and their difference equals the number of months in a year. What are the two numbers?

13. What number is it to which if we add $2\frac{3}{5}$ times itself the sum is equal to the number of feet in 4 rods?

14. Mr. William's little girl is $\frac{1}{6}$ as old as he, and the sum of their ages is 54 years. How old is each?

EXPLANATION OF PROCESSES.

ADDITION.

Find the sum of the following polynomials.

$3x^2 - 2y^2 - 4xy$, $5x^2 - y^2 + 2xy$ and $3xy - 3z^2 - 2y^2$.

$$\begin{array}{rrrr} 3x^2 & -4xy & -2y^2 & \\ 5x^2 & +2xy & -y^2 & \\ & +3xy & -2y^2 & -3z^2 \\ \hline 8x^2 & +xy & -5y^2 & -3z^2 \end{array}$$

DIRECTIONS.—(*a*). Write the quantities in the order of the alphabet simply for convenience. Highest power first.

(*b*). Place similar terms in the same column.

(*c*). Give to each term its proper sign.

(*d*). Add the terms at the left first.

(*e*). $3x^2$ and $5x^2$ equal $8x^2$ just as 3 hats and 5 hats equal 8 hats.

(*f*). Terms having the same letters are not similar terms unless the powers of the letters are also the same *e. g.* $3x^2y$ and $3xy$ are not similar as the power of x is not the same in each.

(*g*). The coefficients of similar terms are not necessarily equal.

(*h*) Remember the algebraic sum is the difference between the sums of the positive and negative quantities *e. g.* $+3xy + 2xy = +5xy$ and $+5xy - 4xy = +xy$ the algebraic sum of the second terms. The third terms $= -5y^2$ there being no positive term.

SUBTRACTION.

From the polynomial $4x^2y + 2xy - 7y + z$ take the polynomial $-2x^2y + 4xy - 3y + z - z^2$.

$$\begin{array}{r}
4x^2y + 2xy - 7y + z \\
-2x^2y + 4xy - 3y + z - z^2 \\
\hline
6x^2y - 2xy - 4y + z^2
\end{array}$$

DIRECTIONS.—(*a*). Write the quantities in the order of the alphabet simply for convenience. Highest power first.

(*b*). Place similar terms in the same column.

(*c*). Give to each term its proper sign.

(*d*). Subtract the terms at the left first.

(*e*). Think of the sign in the subtrahend as changed and find the Algebraic sums *e. g.* in the first term, $-2x^2y$ becomes $+2x^2y$ and the algebraic sum is $6x^2y$. In the second term $+4xy$ becomes $-4xy$ and the algebraic sum is $-2xy$, etc., etc. Since there is nothing from which to subtract $-z^2$ in the last term, it being the *subtrahend*, we bring it down with its sign changed.

MULTIPLICATION.

Multiply the trinomial $3x^2 + 4xy + y^2$ by the binomial $2x - 5y$.

$$\begin{array}{lllll}
3x^2 & + 4xy & + y^2 & & \\
2x & - 5y & & & \\
\hline
6x^3 & + 8x^2y & + 2xy^2 & & \\
 & - 15x^2y & - 20xy^2 & - 5y^3 & \\
\hline
6x^3 & - 7x^2y & - 18xy^2 & - 5y^3 &
\end{array}$$

DIRECTIONS.—(*a*). It will be found convenient to arrange the terms in the order of the alphabet. Highest power first.

(*b*). Write the first term of the multiplier under the first term of the multiplicand.

(*c*). Multiply each term of the multiplicand by the first term (to the left) of the multiplier obtaining the first partial produ:t.

(*d*). Next multiply each term of the multiplicand by the second term of the multiplier obtaining the second partial product and write similar terms in the same column.

(*e*). Find the algebraic sum of the partial products.

(*a*).	(*b*).	(*c*).	(*d*).
$+4x$	$-4x$	$+4x$	$-4x$
$+3$	$+3$	-3	-3

Read carefully.

I. In example (*a*) $+3$ tells us there are to be three equal quantities taken and the product is to be taken additively. Hence the answer is $+12x$.

II. In Example (*b*) $+3$ tells us there are to be three equal quantities taken and the product is to be taken additively. Hence the answer is $-12x$, because in addition we do not change any signs.

III. In Example (*c*) -3 tells us there are to be three equal quantities taken and the product is to be taken *subtractively*. Hence the answer is $-12x$, because $+4x \times 3$ equals $+12x$, and to take $+12x$ *subtractively* makes $+12x$ the subtrahend; therefore, we must change its sign to $-12x$.

IV. In Example (*d*) -3 tells us there are to be three equal quantities taken and the product is to be taken subtractively. Hence the answer is $+12+$ because $-12x \times 3$ equals $-12x$, and to take $-12x$ *subtractively* makes $-12x$ the subtrahend; therefore, we must change its sign to $+12x$.

DIVISION.

Divide the trinomial $6\,x^3 - 5\,x^2y - 6\,x^2y^2$ by the binomial $3\,x + 2y$.

$$
\begin{array}{l|l}
6\,x^3 - 5\,x^2y - 6\,x^2y^2 & 3\,x + 2y \\
\underline{6\,x^3 + 4\,x^2y} & \overline{2\,x - 3\,xy} \\
\quad -9\,x^2y - 6\,x^2y^2 & \\
\quad \underline{-9\,x^2y - 6\,x^2y^2} &
\end{array}
$$

DIRECTIONS.—(*a*.) Write the quantities in the order of the alphabet.

(*b*). Write the divisor at the right of the dividend.

(*c*). Write the quotient under the divisor.

(*d*). Divide the first term of the dividend by the first term of the divisor.

(*e*). Multiply each term of the divisor by each term of the quotient as soon as found, writing the product under similar terms.

(*f*). Algebraic division is finding how many times and in what manner one of the algebraic quantities must be taken to produce the other.

(*g*). How many times and in what manner must $3\,x$ be taken to produce $6\,x^3$? We see by inspection the answer is plus $2\,x$ times. Multiplying each term of the divisor by this we obtain $6\,x^3 + 4\,x^2y$: Subtracting our remainder is $-9\,x^2y$. Bringing down the next term of the dividend $-6\,x^2y^2$ our new dividend becomes $-9\,x^2y - 6\,x^2y^2$, with which we proceed as before. $3\,x$ must be taken $-3\,xy$ times to produce $-9\,x^2y$. Hence the second term of the quotient is $-3\,xy$. Multiplying both terms of the divisor by $-3\,xy$ we obtain $-9\,x^2y - 6\,x^2y^2$ and nothing remains after subtracting.

ANSWERS.

Abbreviations: H. = horizontal line. V. = vertical line. O. = oblique line. P. = perimeter. Prod. = product.

1. $x + y$
2. $x - y$.
3. $x + x$; $2x$.
4. $\frac{z}{y}$.
5. $\frac{y}{z}$.
6. $\frac{x+z}{2}$ or $\frac{1}{2}$ of $(x + z)$.
7. Sum of H.
8. Longer H.—shorter H.
9. Longer H. and V.
10. Answer to Ex. 4.
11. Answer to Ex. 5.
12. Answer to Ex. 6.
13. $x + y$.
14. $x - y$.
15. xy.
16. $x \div y$ or $\frac{x}{y}$.
17. $\frac{1}{2}(x + y)$ or $\frac{x+y}{2}$.
18. $\frac{x}{2} + y$.
19. $(\frac{1}{2}$ of $x) + y$.
20. $x + y + x + y$; $2x + 2y$; $2(x + y)$.
21. $\frac{y}{2x + 2y}$.
22. $\frac{x}{2x + 2y}$.
23. $x + y$ or $\frac{2x + 2y}{2}$ or $\frac{2(x + y)}{2}$, etc.
24. P. of Fig. 1.
25. P. of Fig. 1.
26. The part y is of P.
27. $\frac{1}{3}$ of P.
28. $\frac{1}{4}$ of P.
29. P. of Fig. 1.
30. $\frac{2x + 2y}{6}$ or $\frac{x+y}{3}$, etc.
31. xy.
32. $\frac{1}{6}$ P. of Fig. 1.
33. Area of Fig. 1.
34. $\frac{x+y}{4}$, etc.
35. $\frac{1}{2}$ area of Fig. 1.
36. $\frac{xy}{3}$.
37. $(x + y) + (x + z)$.
38. $4(x + z)$.
39. Answer to Ex. 37.
40. $2x + 2x$; $4x$; $x + x + x + x$.
41. A square.
42. See note.

43. x times x.
44. See note.
45. 1.
46. 1. 2.
47. Equals their sum.
48. x^3. x^4. x^5.
49. Its area.
50. One side.
51. $2x$ or $\frac{4x}{2}$ or $\frac{x+x}{2}$.
52. x, etc.
53. $\frac{4x}{8}$ or $\frac{2x}{4}$ or $\frac{x}{2}$.
54. $\frac{1}{16}$; $\frac{1}{16}$; $\frac{1}{4}$.
55. $\frac{3}{4}$.
56. $\frac{3}{8}$.
57. x^2; y^2; z^2.
58. x^3; y^3; z^3.
59. y; z.
60. 1. $2y$; $3xy$.
61. 2. $y+z$; $+xy+z$.
62. 3. $2+x+z$; $2x+y+z$.
63. Binomial polynomial.
64. $2x+y$.
65. $x+x+y$.
66. $z-(x+y)$.
67. See note.
68. $z-(x-y)$.
69. Coefficient; Exponent.
70. 2; 3; 2; 1.
71. 6; 5; 8; 1.
72. $x+y$.
73. $x-y$.
74. xy.
75. $\frac{x}{y}$.
76. $\frac{y}{x}$.
77. $(x+y)^2$.
78. (x^2+y^2).
79. $(x+y)^3$.
80. (x^3+y^3).
81. $(x+y)(x-y)$.
82. $xy(x-y)$.
83. $\frac{(x+y)}{(x-y)}$.
84. $4+x$.
85. $4x$.
86. Their sum.
87. Their product; the product of their sum and difference.
88. $x+y$.
89. $4+x$.
90. $4x$.
91. $x-2$.
92. $\frac{1}{y}$ of x.
93. $\frac{x}{y}$.
94. $2x+2y$, etc.
95. xy.
96. y^2.
97. $x-50$.
98. $15a$.
99. $50-c$.

100. $3x + \frac{5y}{100}$.
101. $3y$ or $8y - 5y$.
102. $x - \frac{5y}{3}$.
103. xy.
104. $\frac{500}{y}$ or $\frac{1}{y}$ of 500.
105. $\left(\frac{5y}{6}\right) 10$ or $\left(\frac{50y}{6}\right)$.
106. $\frac{yz}{x}$.
107. $(b+c)\,d$, or $(bd+cd)$.
108. $\frac{200}{x}$.
109. xy.
110. $(2x+2y)\,z$ or $(2xz+2yz)$ or $z\,(x+y)\,2$.
111. x^2.
112. $\frac{y}{8}$.
113. $(z-y)\,x$ or $(xz-xy)$.
114. $18x$.
115. $25x$.
116. $\frac{xy}{z}$ or $\frac{1}{z}$ of xy.
117. $8+x$.
118. $4+x$.
119. $x+y+\sqrt{x^2+y^2}$.
120. $\frac{x+y+\sqrt{x^2+y^2}}{2}$.
121. $2x+y$.
122. Triangle.
123. $x\left(\frac{1}{2}y\right)$; $\frac{x\left(\frac{1}{2}y\right)}{2}$.
124. Answer to Ex. 119.
125. $a+b+\sqrt{a^2+b^2}$.
126. $12x$.
127. $6x^2$; $3x^2$; $\frac{6x^2}{4}$ or $\frac{3x^2}{2}$.
128. $\sqrt{25x^2-16y^2}$.
129. Negative.
130. Negative.
131. $+8-5-2+15+80-16$.
132. $+5+2-3-2+8-6$.
133. See solution Ex. 134.
134. See solution Ex. 134.
135. The difference between positive and negative quantities.
136. $+6x$.
137. $-6x$.
138. $+10x$.
139. $-10x$.
140. $-4x$.
141. 0.
142. $+65x$.
143. $-16ab$.
144. $+107z^2$.
145. $+36x$.
146. 0.
147. $294cd$.
148. 0.
149. $x+y-z$.
150. $2x-y$.
151. $-x-2y$.
152. xy.
153. $a+b-c$.

154. 0.
155. $5xy$.
156. 0.
157. 0.
158. 0.
159. 0.
(a). $2ax + 8xy - n + am$.
(b). $xyz + 23xy^2 - 9abc - 9axy$.
(c). $x^2y^2 - 14xy + ax$.
(d). $4x^2b + 5xbc - 17b^2c + 5c^3$.
160. An equation.
161.*$3 + 4 = 7$ (2d).
162.*$6 - 2 = 4$ (2d).
163.*$12 + 4 = 16$ (2d).
164.*$8 = 3 + 5$ (1st).
165.*$8 = 8$ (1st).
166.*$10 = 10$ (2d).
167. 5.
168. 8.
169. 10.
170. See solution.
171. 8 and 16.
172. 3 and 15.
173. 3.
174. 20 and 60.
175. 15 and 45.
176. 4 and 8.
177. 6.
178. 160 and 640.
179. 12 and 120.
180. 6; 12; 18.
181. Added 2 to each member.
182. No.
183. $12 = 12$.
184. No.
185. $4 = 4$.
186. No.
187. $3 = 3$.
188. No.
189. No.
190. Both; equation; equality.
191. Both; equal; destroyed.
192. 32.
193. 13.
194. 20.
195. 98.
196. 24; 18.
197. 16; 25.
198. 7.
199. Added $+x - 8 + 4$ to both members.
200. 4; 2; 4;
201. 4.
202. $\frac{4}{11}$.
203. 3.
204. 10; 30; 8.
205. 20; 40; 60.
206. 7.
207. 4; 16.
208. 10.

*The answers here are but one of many that may be given.

209. 20; 35.
210. 8.
211. $\frac{4}{5}$.
212. 8.
213. 7.75; 8.25.
214. 3; 4.
215. Multiply both members by 3.
216. 10; 7.50; 3.75.
217. 16; 32; 12.
218. 24.
219. 60.
220. 8.
221. 18.
222. $\frac{5}{8}$.
223. 11.
224. 20; 30.
225. Divided the product of the extremes by the product of the means.
226. 2; 3.
227. 12; 6.
228. 27; 9.
229. 20; 30; 120.
230. 36; 12.
231. 8; 4; 12.
232. 4; 2; 1.
233. 30; 52.
234. 21.
235. $+3x$.
236. 0.
237. $a+b-c$.
238. $x+y-z$.
239. xy.
240. $x-y$.
241. The same.
242. Changed the signs of subtrahend and added.
243. Any numbers.
244. Change the signs of the subtrahend and get the algebraic sum.
245. $x-y+z$.
246. The same.
247. Changed the signs of the subtrahend and found the algebraic sum.
248. Subtrahend; sum.
249. $z-x-y$.
250. $4x$.
251. $5y$.
252. $-11y$.
253. $11x$.
254. $8xy$.
255. $4xy$.
256. $13xyz$.
257. $12x$.
258. $-3x^2$.
259. $18x^2y^2$.
260. Given.
261. Given.
262. $2x+2y$.
263. $x+3y$.
264. $2y$.

265. y.
266. $2x+y+z$.
267. $2x$.
268. $2x-2y$.
269. z.
270. $2x+2y+2z$.
271. $x+2y+2z$.
272. $x+y+z$.
273. $2z$.
274. $2x+y+3z$.
275. $3x+y+2z$.
276. $4x+2z$.
277. $20x+16yz+4y+16z$; $16x+16yz+8z$.
278. $4x+4y+8z$.
279. $14x+8yz+6y+16z$.
280. Given.
281. $z-y$.
282. Length of 45.
283. No. 23 is $+y$ not $-y$.
284. 0. Add y.
285. Length of 23.
286. y.
287. No difference.
288. $+y$; $-y$.
289. $-z+y$.
290. Parenthesis; changed.
290*a*. Terms; parenthesis; changed.
290*b*. Parenthesis; parenthesis; before.
(*a*). $x-(y+z)$; $x-y-z$.
(*b*). $z-(x-y)$; $z-x+y$.
(*c*). $z+(x+y)$; $z+x+y$.
(*d*). $z+(x-y)$; $z+x-y$.
291. $2+y-z$.
292. $5x-a-b$.
293. $7y-x-2y$.
294. $7y-x+2y$.
295. $7y+x-2y$.
296. $3x-6y-z$.
297. $x+(y-z)$.
298. $x-(y-z)$.
299. $x-(y+z)$.
300. $2x-(y+2z)$.
300*a*. $+$; $-$;
301. $-20x$.
302. $+64x$.
303. $-42x$.
304. $+63x$.
305. $-30x$.
306. $-96y$.
307. $-36y$.
308. $+21y$.
309. $+8y$.
310. $+81y$.
311. $-36x-9y$.
312. $16x-24y$.
313. $-63x+56y$.
314. x^2y.
315 $-x^2yz$.
316. x^2-xy.
317. x^2-xy.
318. $4x+2y$, etc.
319. $2x+y$.
320. x^2+xy; $x(x+y)$.
321. $4x-2y$, etc.

322. $2x-y$, etc.

323. $\frac{2x-y}{2}$ or $\frac{4x-2y}{4}$.

324. x^2-xy or $x(x-y)$.

325. $4x+4y$, or $x(x-y)$. etc.

326. $x+y$.

327. $\frac{x+y}{2}$.

328. $\frac{1}{16}$.

329. $x^2+2xy+y^2$.

330. The square of the sum of 2 quantities.

331. $a^2+2ab+b^2$.

332. $y^2+2yz+z^2$.

333. $4x^2+8xy+4y^2$; $4a^2+8ab+4b^2$.

334. $x^2+2xy+y^2$.

335. $y^2+2yz+z^2$.

336. $a^2+2ab+b^2$.

337. $b^2+2bc+c^2$.

338. $4x^2+8xy+4y^2$.

339. $4a^2+8ab+4b^2$.

340. $4c^2+8cd+4d^2$.

341. $9x^2+12xy+4y^2$.

342. $16x^2+24xy+9y^2$.

343. $25x^2+50xy+25y^2$.

344. $a^2b^2+2ab^2c+b^2c^2$.

345. First; first; second; second.

346. $4x-4y$ or $4(x-y)$.

347. $2x-2y$.

348. $\frac{3x-3y}{2}$ or $\frac{3(x-y)}{2}$.

349. $x^2-2xy+y^2$.

350. $a^2-2ab+b^2$.

351. $4x^2-8xy+4y^2$.

352. The square of the difference of 2 numbers.

353. $x^2-2xy+y^2$.

354. $4x^2-8xy+4y^2$.

355. $4a^2-8ab+4b^2$.

356. $4b^2-8bc+4c^2$.

357. $25x^2-50xy+25y^2$.

358. $16y^2-32yz+16z^2$.

359. $9a^2-18ab+9b^2$.

360. $36z^2-72zy+36y^2$.

360*a*. First; first; second; second.

361. $4x$.

362. $2x+2y$.

363. $2x$.

364. $2x-2y$.

365. x^2-y^2.

366. The difference of the squares of 2 numbers.

367. a^2-b^2.

368. $4x^2-4y^2$.

369. The product of the sum and difference of 2 numbers.

370. y^2-z^2.

371. $4y^2-4z^2$.

372. $16a^2-4b^2$.

373. $16x^2-4y^2$.

374. $25x^2-25z^2$.

375. $36a^2 - 16b^2$.
376. $49x^2 - 9y^2$.
376*a*. Difference; squares.
377. $3x + 11$.
378. $3x - 2$.
379. $3x - 10$.
380. $4x - 5$.
381. $x^2 + 7x + 12$.
382. $x^2 + x - 12$.
383. $x^2 - 7x + 12$.
384. x.
385. x^2.
386. It is the same.
387. 7.
388. $7x$; the same.
389. 12; the same.
389*a*. Square; common; sum; unlike; product.
390. $x^2 + 7x + 10$.
391. $x^2 + x - 30$.
392. $x^2 - 17x + 70$.
393. $a^2 + 5ab + 4b^2$.
394. $y^2 + 15y + 50$.
395. $y^2 + (yz - 3y) - 3z$.
396. $z^2 - 2z - 8$.
397. $a^2 + a(b + c) + bc$.
398. $4z^2 - 4z - 21$.
399. $a^2 - 10a + 21$.
400. $4x^2 + 12x + 8$; $x^2 - y^2$.
401. It does not destroy the equality.
402. $4x = 16 + 4$.
403. 6.
404. 16.
405. 20.
406. 34.
407. 6.
408. 240.
409. 16; 80.
410. 12; 36.
411. 30; 10.
412. $2\frac{3}{8}$.
413. $\frac{6}{17}$.
414. $1\frac{37}{105}$.
414*a*. + ; − .

Review Exercise—

(1). $x + 2z$.
(2). $x - 2y$.
(3). $+5x^2 - 9xy + 5y^2$.
(4). $x - 4y = 21 + 6$.
(5). $x^2 + 6x + 9$.
(6). $x^2 + 5x + 6$.
(7). $4x^2 - 8xy + 4y^2$.
(8). 9; 12.
(9). 13.
(10). $x^3 + y^3$.

415. $2x$.
416. $-3y$.
417. $3x^2$.
418. $-5x$.
419. $6x^2y$.
420. $-3y^2$.
421. $2x^3$.
422. $-8xy^2$.
423. $7y^3$.
424. $-12xyz$.
425. $8ab$.
426. $x + y$.
427. $x - y$.
428. $x + 3$.
429. $x + 4$.
430. $x + y$.
431. $x - y$.
432. $2x + y$.

433. $2x + 3y$.
434. $x^2 - xy + y$.
435. $3x - 2y$.
436. $8x + 4y$.
437. $2x + 3y$.
438. $6x$.
439. $6x + 6z$.
440. $5x + 5y$; $5x - 5y$.
441. $2y + 4z$.
442. $x^3 + x^2y + xy^2 + y^3$.
443. $x^4 - 2x^3y + 4x^2y^2 - 8xy^3 + 16y^4$.
444. $x^2 + xy + y^2$.
445. It does not destroy the equality.
446. 4; 8.
447. They are equal.
448. Added equal quantities to both members.
449. $2x = 8$.
450. $y = 8$.
451. 3; 2.
452. Equal; unlike; added; equal alike; subtracted.
453. The same; unlike; like.
454. Yes; by multiplying.
455. Multiplied by 2.
456. Divided both members by 2.
457. 20; 15.
458. 3; 5.
459. 10; 5.
460. 15; 45.
461. 12; 48.
462. 17; 85.
463. 59; 27.
464. $\frac{5}{12}$.
465. $\frac{3}{4}$.
466. 6; 12.
467. 72.
468. $\frac{9}{10}$.
469. 72.
470. 2; 4.
471. 100; 20.
472. 4; 3.
473. 48; 28.
474. 20 more.
475. 28; 12.
476. 30.
477. 1250; 250.
478. $\frac{1}{3}$; $\frac{2}{3}$.
479. 6; 5.
480. 8; 2.
481. 24; 12.
482. 33; 22.
482*a*. 6; 4.
483. Quantity; factors.
(3). $ab\,(ab - b + a)$.
(4). $y\,(y^2 + 3y - 2)$.
(5). $3y\,(y - 2z + 3z^2)$.
484. x and x.
485. x and y.
486. $(x + y)\,(x - y)$.
487. $(z - y)\,(z + y)$.
488. $(x + y)\,(x + y)$.
489. $(x - y)\,(x - y)$.
490. $(x + 4)\,(x + 4)$.
491. $(3x - y)\,(3x - y)$.
492. $3x^2z^3\,(4x^2y - 5xz - 2b)$.
493. z and z.
494. x; y; and z.
495. $(a + b)\,(a - b)$.
496. $(2a + 3b)\,(2a - 3b)$.

497. $(a-3)(a-5)$.
498. $(x+3)(x+4)$.
499. $(x+4)(x-3)$.
500. $(x-3)(x-4)$.
501. $6xy(y-2xy^2-6)$.
502. $12(12a^2+22ab+11b^2)$.
503. Factors of x^2-y^2.
504. Factors of $x^2-(y+z)^2$.
505. $x+y+z$ and $x-y-z$.
506. $a+(b+c)\,a-(b+c)$.
507. $2x+(x+y)\,2x-(x+y)$.
508. $x(x+y)$.
509. $(x^2+y^2)(x^2-y^2)$.
510. $z+(x+y)\,z-(x+y)$.
511. $(x+y)+z\,(x+y)-z$.
512. $(xy+yz)(xy-yz)$.
513. $(x+a)(x+b)$.
514. $(x-a)(x-b)$.
515. $(x-y)(x-z)$.
516. $(x+y+x-y)(x+y-x+y)$.
517. $3a^2c^3(4a^2b-5ac-2d)$.
518. Square.
519. $x^2+2xy+y^2$.
520. $5x^2+10xy+5y^2$.
521. $5(x+y)(x+y)$.
522. $9(a+b)(a+b)$.
523. $8(y+z)(y+z)$.
524. A rectangle, or oblong.
525. x^2+5x+6.
526. $2x^2+10x+12$.
527. $2(x+3)(x+2)$.
528. $5(x+2)(x-3)$.
529. $8(x+3)(x+2)$.
530. $7(x-3)(x+2)$.
531. $6(x-y)(x-y)$.
532. $5(x+y)(x-y)$.
533. $2(x-6)(x-4)$.
534. $7a^2b(2abc+b^2c-3d)$.
535. $5[x^2-(a+b)^2]$ or $5(x+a+b)(x-a-b)$.
536. $16(a+b)(a+b)$.
537. A cube.
538. $x^3+3x^2y+3xy^2+y^3$.
539. 27.
540. Cube; 3 times; the square; 3 times; first; square; cube.
541. $a^3+3a^2b+3ab^2+b^3$.
542. $y^3+6y^2+12y+8$.
543. $8+12z+6z^2+z^3$.
544. $x^3-3x^2y+3xy^2-y^3$.
545. Same as the cube of the sum only; the second and last terms are minus.
546. $a^3-3a^2b+3ab^2-b^3$.
547. $8x^3+12x^3y+6x^3y^2+x^3y^3$.
548. $(x+y)^3$ or $(x+y)(x+y)(x+y)$.

549. $(x-y)^3$.
550. $(2+z)^3$.
551. $(2x^2-3y^2)^3$.
552. $2(x+y)^3$.
553. $4(x-y)^3$.
554. $8(x-y)^3$.
555. $6(x+y)^3$.
556. $2(a+x)$.
557. $3(a+x)$.
558. $x(a+x)$.
559. $y(a+x)$.
560. $5(a+x)$.
561. $(x+y)(a+x)$.
562. $5(a+x)$.
563. $(x+y)(a+x)$.
564. $(a+b)(a+x)$.
565. $(a+x)(y-z)$.
566. $(x^2-4)(y+z)$ or $(x+2)(x-2)(y+z)$.
567. $(b-d)(a-c)$.
568. $(a+1)(a-1)(x-1)$.
569. $(a+b)(x+y)(x-y)$.
570. $(5b+3a)(a-c)$.
571. $(a^2+x^2)(b+c)(b-c)$.

Review Exercise—
(1). $(5+2x)^2$.
(2). $(3x-5)^2$.
(3). $(3x-y)^2$.
(4). $(2x-3y)^2$.
(5). $(2+y^3)$.
(6). $(5x+1)(5x+6)$.
(7). $(z+9)(z+2)$.
(8). $4(x-y)(x+y)$.
(9). $x+y+z(x-y-z)$.
(10). $(ab+bc)(ab-bc)$.

572. Divide; numerator; denominator; same.
572*a*. Multiplied; number; not.
573. $\frac{1}{2}$.
574. $\frac{x+y}{2x+2y}$.
575. $\frac{1}{2}$.
576. $25xyz$.
577. $15x^2y^2$.
578. $\frac{15x^2y^2}{25xyz}$.
579. $\frac{3xy}{5z}$.
580. $\frac{x}{y}$.
581. $\frac{3y^7}{17}$.
582. $\frac{x+y}{4(x-y)}$.
583. $\frac{x-y}{3(x+y)}$.
584. $\frac{x-1}{x+1}$.
585. $\frac{1}{x-y}$.
586. $\frac{x^2-y^2}{x^2+y^2}$.
587. $\frac{x^2-y^2}{(x-y)^2}$.

588. $\frac{4x^2}{2x(x+y)}$.

589. $\frac{x+y}{(x+y)^2}$.

590. $\frac{x^2-y^2}{(x+y)(2x-y)}$

590*a*. Denominator; numerator.

591. $\frac{xy-y}{x}$.

592. $\frac{3x+3y}{2}$.

593. $\frac{x+y}{2}$.

594. $\frac{x^2+3xy+y^2}{x+y}$.

595. $\frac{x^2-y^2+1}{x+y}$.

596. $\frac{2y^2+x}{y}$.

597. $\frac{z^2-y^2}{x-y}$. 598. $\frac{x^2-7}{x-5}$.

599. $\frac{xz+y^2}{yz}$. 600. $\frac{xz-y^2}{yz}$.

601. $\frac{(x^2+xy)+(xz+yz)}{x^2-y^2}$.

602. $\frac{(x^2-xy)-(xz+yz)}{x^2-y^2}$.

603. $\frac{x+y}{x^2-7x+12}$.

604. 1. 605. $\frac{13y}{12}$.

606. $\frac{2xy+2y^2}{x^2-y^2}$, or $\frac{2y}{x-y}$.

607. $2+\frac{64x}{21}$.

608. $\frac{x^2-3x+xy-4y}{x^2-7x+12}$.

609. Numerator; dividing; multiplies.

610. Multiplying; dividing.

611. $\frac{xz}{y}$.

612. $\frac{x^2-x-12}{z}$.

613. $\frac{x^2-12x+32}{y}$.

614. $\frac{x^2+2xy+y^2}{x-y}$.

615. $\frac{x^2-5x-24}{xyz}$.

616. $\frac{x}{y^2}$. 617. $\frac{x+y}{x-y}$.

618. $\frac{x^2+y^2}{x+y}$.

619. $\frac{x^2-2xy+y^2}{xz}$.

620. $\frac{4x^2+8xy+4y^2}{x+y^2}$.

621. $\frac{x-y}{z}$.

622. $\frac{xy}{y^2+7y+12}$.

623. $\frac{x+y}{z^2+3z-28}$.

624. $\frac{y+z}{xy}$.

625. $\frac{x^2y^2}{z^2-12z+32}$.

626. $\frac{x-y}{3}$. 627. $\frac{ac}{bd}$

628. $\frac{x^2-2xy+y^2}{x^2+2xy+y^2}$.

629. $\frac{zb}{x^2-2xy+y^2}$.

630. $\frac{xy^2z}{y^2-5y-24}$.

631. $\frac{a^2b^2c^2}{x^2+5x+4}$.

632. $\frac{c^4d^4b}{x^2-18x+80}$.

633. Denominator; numerator; divisor.

634. $\frac{bx}{ay}$. 635. $\frac{dz}{cy}$.

636. $\frac{ay}{bx}$. 637. $\frac{ay}{dx}$.

638. $\frac{ax+ay}{bx-by}$.

639. $\frac{32z-4yz}{24c^2xz}$.

640. $\frac{2bxz-by}{2za}$.

641. $\frac{4bx+8b}{2ax+2a}$.

642. $\frac{x^3+x^2}{x^3-x}$.

643. y.

644*a*. 3; 5; 4.

645. 3; 2; 4.

646. 6; 8; 10.

647. 24; 60; 120.

648. 64; 72; 84.

649. 10; 30; 40.
650. Given.
651. 22.
652. 24.
653. 31.
654. 32.
655. 40.
656. 12.
657. 13.
658. 54.
659. 99.
660. 73.
661. 123.
662. .3.
663. .4.
664. .15.
665. .25.
666. .16.
667. .45.
668. .32.
669. 2.64 +.

670. $e+f+g$.

671. $a+b+c+.3+5x^2$.

672. $3x+2+2y$.

673. $5x^2-3xy+4y^2$.

674. Given; $\frac{22x+22y}{7}$.

675. $\frac{11(x+y)(x+y)}{14}$.

676. $\frac{22x-22y}{7}$.

677. $\frac{11(x-y)(x-y)}{14}$.

678. $12(x+y)$.

679. $6(x+y)^2$.

680. $12(x-y)$.

681. $6(x-y)^2$.

682. $4x(5x+6)$.
683. $5(x+y)^2$.
684. $9(x+y)^2$.
685. $\frac{x^2+xy}{2}$.
686. $\frac{19x^2+37xy+18y^2}{2}$.
687. $(12x)^2$.
688. $4x(42x)$.
689. $2(18x^2+75x+50)$.
690. $9x^2+15x+4$.
691. $(x-y)^2$.
692. $4(x+y)(x-y)$.
693. $x^3+3x^2y+3xy^2+y^3$.
694. 6 rods.
695. $8x+3y$.
696. $2x^2+4x+y+4$.
697. $22x+7y$.
698. $8(2x+y)$.
699. $11x^2+15y+21z+4z^2-2y^2$.
700. $-2a^3+3a^2b-8b^2c$.
701. $-xy+3cd-5b^2$.
702. $6x^2y^2-12xy-8xyz-4abc+z$.
703. $a^2b^2+c^2d$.
704. $[a^3+3a^2x+3ax^2+x^3]$.
705. $6x^2-2x-8$.
706. $5x^2+4xy+3y^2$.
707. 7.2. **708.** 14.
709. $40\frac{3}{23}$. **710.** 5.
711. $2x^3+1$.
712. $2x^2-3y^2$.
713. $1+xy$.
714. $xy+yz$.
715. $(a-3)(a-4)$.
716. $(x+4)(x+3)$.
717. $(a+4)(a-3)$.
718. $x-y$.
719. $27x^3+81x^2y+81xy^2+27y^3$.
720. $27x^3-81x^2y+81xy^2-27y^3$.
721. $(2x^2y-3xy^2)^2$.
722. $(xy+5)^2$.
723. 46; 9; 7.
724. $1\frac{7}{8}$.
725. $\frac{77x^2+154xy+77y^2}{2}$.
726. $1\frac{1}{4}$. **727.** 4; 5; 6.
728. 25; 40. **729.** 6; 4.
730. 15; 30. **731.** $\frac{5}{6}$; $\frac{1}{2}$.
732. 100; 60. **733.** 5.
734. $\frac{2y^2}{3x}$.
735. $\frac{2}{3x}$.
736. 8.
737. 5; 3; 1.
738. 74.
739. 0.
740. 38; 39; 40.
741. 16; 44.
742. 48.
743. 46; 32.
744. 18.
745. 12; 18.
746. 3100; 2500; 1800.

747. 160; 480.
748. 35; 20; 50.
749. 26. **750.** 18.
751. 16; 8. **752.** 80.
753. 3. **754.** 2; 4; 10.
755. 36; 48. **756.** $\frac{3}{5}$.

EXERCISE I.

1. 10.	3. 6.	5. 4.	7. 20.	9. 6.
2. 5.	4. 5.	6. 5.	8. 50.	10. 9.

EXERCISE II.

1. 18.	3. 36.	5. 35.	7. 3; 6; 3.	9. 14. 28.
2. 24.	4. 21; 7.	6. 32.	8. 36; 18.	10. 33.

EXERCISE III.

1. 5.	3. 20.	5. 24.	7. 48.	9. 5; 12.
2. 10.	4. $10\frac{3}{7}$.	6. 36.	8. 12; 6.	10. 18.

EXERCISE IV

1. 38; 46.	4. 6; 12.	7. 12; 8. 4.	9. $\frac{4}{5}$.
2. 10; 3.	5. 3; 5.	8. 6; 18.	10. 12; 4.
3. 21 + 32.	6. 21; 25.		

EXERCISE V.

1. $\frac{a+b}{2}$; $\frac{a-b}{2}$.
2. 3; 4.
3. 6 p.m.
4. $\frac{c}{4}$; $3\left(\frac{c}{4}\right)$.
5. $\frac{6a+b}{6}$; $\frac{6a-b}{4}$.
6. 2; 1; 3.
7. 4; 3; 5.
8. 5: 6; 8.
9. 18; 9.
10. $\frac{2b}{3}$; $\frac{b}{3}$.

EXERCISE VI.

1. 1670. 2. 9300. 3. 462.
4. $a-c+b$; $(a-c)$; $c-[b+(a-c)]$.
5. 5; 10; 15. 6. x^2-y^2.
7. $(x-y)(x+y)$.
8. $(z+6)(z-8)$.
9. $\frac{x-y}{x+y}$. 10. 48.

EXERCISE VII.

1. 72.	3. 36.	5. 40.	7. 80.	9. 45.
2. 96 in all.	4. 80.	6. 30.	8. 39.	10. 36.

EXERCISE VIII.

1. $\dfrac{2x^2+14x+23}{x^2+7x+12}$.
2. $\dfrac{2x+7}{x^2+7x+12}$.
3. $\dfrac{2x^2-6x-40}{x^2+6x+8}$.
4. $\dfrac{2(x+12)}{x^2+6x+8}$.
5. $\dfrac{-4xy}{x^2-y^2}$.
6. $\dfrac{2(x^2+y^2)}{x^2-y^2}$.
7. $\dfrac{y+4}{y+2}$.
8. $\dfrac{a}{b}$.
9. $\dfrac{abc-ab}{cd}$.
10. $\dfrac{x+y}{a}$.

EXERCISE IX.

1. $\dfrac{ad+bc}{bd}$.
2. $\dfrac{xy-z}{y}$.
3. $\dfrac{7x}{12}$.
4. $\dfrac{5x}{12}$.
5. $\dfrac{5x}{24}$.
6. $\dfrac{5x+72}{24}$.
7. 0.
8. $\dfrac{y}{12}$; $\dfrac{2y}{12}$.
9. $\dfrac{y}{8}$.
10. $\dfrac{x+y}{2}$.

EXERCISE X.

1. $x+y$.
2. $x-y$.
3. $(y-3)$ by $(y-4)$.
4. $(2x+3)$ by $(2x+4)$.
5. $(2x-2y)$ by $(2x+2y)$.
6. x by y.
7. $(6y+6z)$ by $(6y-6z)$.
8. $3x+2y$.
9. $x+y$.
10. $x^2-2xy+y^2$.

EXERCISE XI.

1. $x+2z$.
2. $x-2y$.
3. $5x^2-9xy+5y^2$.
4. $x-4y=21+6$.
5. $x^2-2xy+y^2$.
6. 33; 25.
7. 6.
8. 38.
9. y^3-3y^2+2z-6.
10. $140\frac{1}{3}$; $176\frac{1}{3}$.
11. $8a$; $16b$; $100c$; $10d$; $\dfrac{e}{100}$.
12. 22; 10.
13. 18.
14. 45; 9.